MATH 3160: Quiz #5 – SOLUTIONS

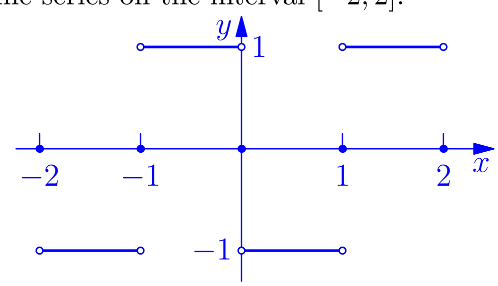
/10 **Problem 1:** Consider the function $f(x) = \begin{cases} -1 & 0 \le x < 1 \\ 1 & 1 \le x < 2 \end{cases}$.

(a) Find the sine series for $f(x)$.

The sine series representation of f is $f(x) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{2}\right)$ where

$$B_n = \frac{2}{2}\int_0^2 f(x)\sin\left(\frac{n\pi x}{2}\right)\,dx = \int_0^1 (-1)\sin\left(\frac{n\pi x}{2}\right)\,dx + \int_1^2 (1)\sin\left(\frac{n\pi x}{2}\right)\,dx$$
$$= -\frac{2}{n\pi}\left[-\cos\left(\frac{n\pi x}{2}\right)\right]_0^1 + \frac{2}{n\pi}\left[-\cos\left(\frac{n\pi x}{2}\right)\right]_1^2$$
$$= \boxed{\frac{2}{n\pi}\left[2\cos\left(\frac{n\pi}{2}\right) - 1 - (-1)^n\right]}$$

(b) Sketch the graph of your sine series on the interval $[-2, 2]$.



(c) Find the cosine series for $f(x)$.

The sine series representation of f is $f(x) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi x}{2}\right)$ where

$$A_0 = \int_0^2 f(x)\,dx = 0$$

and

$$A_n = \frac{2}{2}\int_0^2 f(x)\cos\left(\frac{n\pi x}{2}\right)\,dx = \int_0^1 (-1)\cos\left(\frac{n\pi x}{2}\right)\,dx + \int_1^2 (1)\cos\left(\frac{n\pi x}{2}\right)\,dx$$
$$= -\frac{2}{n\pi}\left[\sin\left(\frac{n\pi x}{2}\right)\right]_0^1 + \frac{2}{n\pi}\left[\sin\left(\frac{n\pi x}{2}\right)\right]_1^2$$
$$= \boxed{-\frac{4}{n\pi}\sin\left(\frac{n\pi}{2}\right)}$$

(d) Sketch the graph of your cosine series on the interval $[-2, 2]$.