MATH 316: Quiz #4 – SOLUTIONS

/5 **Problem 1:** Solve the following (wave equation) initial boundary value problem:

$$u_{tt} = c^2 u_{xx} \qquad x \in [0,1], \quad t \in [0,\infty)$$
$$u(0,t) = u(1,t) = 0$$
$$u(x,0) = 0.01 \sin 3\pi x, \qquad u_t(x,0) = 0.$$

For the given boundary conditions we have the general solution

$$u(x,t) = \sum_{n=1}^{\infty} \sin(n\pi x)\big[A_n \cos(n\pi ct) + B_n \sin(n\pi ct)\big].$$

Applying the initial conditions we have:

$$u_t(x,0) = 0 = \sum_{n=1}^{\infty} \sin(n\pi x)\Big[n\pi c B_n\Big] \implies B_n = 0 \quad \forall n$$

$$u(x,0) = 0.01\sin(3\pi x) = \sum_{n=1}^{\infty} A_n \sin(n\pi x) \implies \begin{cases} A_3 = 0.01 \\ A_n = 0, \qquad n \neq 3 \end{cases} \quad \text{(by inspection)}$$

$$\implies \boxed{u(x,t) = 0.01\sin(3\pi x)\cos(3\pi ct)}$$

/5 **Problem 2:** Let $A, B, C \in \mathbb{R}$ be given non-zero constants. Without actually solving the equations, show that the following two initial value problems must have the same solution for $t \geq 0$:

(a) $\begin{cases} Ay'' + By' + Cy = 0 \\ y(0) = 0, \quad y'(0) = 1 \end{cases}$ (b) $\begin{cases} Ay'' + By' + Cy = A\delta(t) \\ y(0) = 0, \quad y'(0) = 0 \end{cases}$

$$A(s^2Y_1 - 1) + B(sY_1) + C(Y_1) = 0$$
$$\implies Y_1(s) = \frac{A}{As^2 + Bs + C} \equiv G(s)$$
$$\implies y_1(t) = g(t)$$

$$A(s^2Y_2) + B(sY_2) + C(Y_2) = A$$
$$\implies Y_2(s) = \frac{A}{As^2 + Bs + C} = G(s)$$
$$\implies y_2(t) = g(t)$$

Thus: $y_1(t) = y_2(t) = g(t)$.