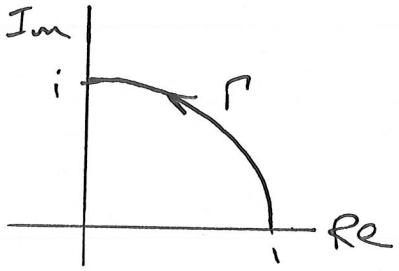
Name: SOLUTIONS Student #: _____ /10

MATH 3000: Quiz #4

/5 **Problem 1:** Evaluate $\int_\Gamma \bar{z}\,dz$ where Γ is the part of the unit circle in the first quadrant.



parametrize Γ: $z = e^{i\theta}$, $\theta \in [0, \frac{\pi}{2}]$

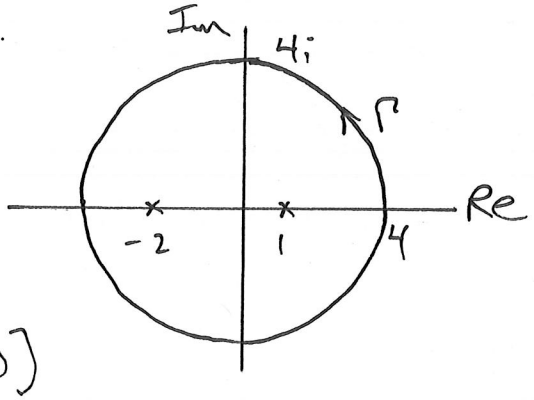
$$dz = ie^{i\theta}d\theta$$

$$\bar{z} = e^{-i\theta}$$

$$\rightarrow \int_\Gamma \bar{z}\,dz = \int_0^{\pi/2} (e^{-i\theta})\, ie^{i\theta}\,d\theta = i\int_0^{\pi/2} d\theta$$

$$= \boxed{i\frac{\pi}{2}}$$

/5 **Problem 2:** Evaluate $\int_\Gamma \frac{z}{(z+2)(z-1)}\,dz$ where Γ is the circle $|z| = 4$.



Γ encloses poles at $z = 1, -2$ so

$$\int_\Gamma f\,dz = 2\pi i\left[\text{Res}(1) + \text{Res}(-2)\right]$$

$$= 2\pi i\left[\lim_{z\to 1}(z-1)f(z) + \lim_{z\to -2}(z+2)f(z)\right]$$

$$= 2\pi i\left[\lim_{z\to 1}\frac{z}{z+2} + \lim_{z\to -2}\frac{z}{z-1}\right]$$

$$= 2\pi i\left[\frac{1}{3} + \frac{-2}{-3}\right] = \boxed{2\pi i}$$