MATH 3000: Quiz #1 - SOLUTIONS

/5 **Problem 1:** Solve: $(2-i)z + 8z^2 = 0.$

$$z(2-i+8z) = 0 \implies z = 0 \text{ or } z = \frac{i-2}{8} = -\frac{1}{4} + \frac{1}{8}i$$

/5 **Problem 2:** Show that $|e^z| < 1$ if $\operatorname{Re} z < 0$.

Let z = x + iy, then:

$$|e^{z}| = |e^{x+iy}| = |e^{x}e^{iy}| = |e^{x}|\underbrace{|e^{iy}|}_{1} = e^{x} < 1$$

since x < 0.

Alternatively:

$$\operatorname{Re} z < 0 \implies |e^z| = \sqrt{e^z \overline{e^z}} = \sqrt{e^z e^{\overline{z}}} = \sqrt{e^{z + \overline{z}}} = \sqrt{e^{2\operatorname{Re} z}} < \sqrt{e^0} = 1$$