MATH 1300: Quiz #7 – SOLUTIONS

/5 **Problem 1:** Solve the following linear system: $\begin{cases} (1+i)x + 2y = 3 \\ x - iy = 1 \end{cases}$

$$\begin{bmatrix} 1+i & 2 \\ 1 & -i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \implies \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1+i & 2 \\ 1 & -i \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \frac{1}{(1+i)(-i)-2} \begin{bmatrix} -i & -2 \\ -1 & 1+i \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \frac{1}{-1-i} \begin{bmatrix} -3i-2 \\ -2+i \end{bmatrix}$$

$$= \frac{-1+i}{2} \begin{bmatrix} -3i-2 \\ -2+i \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 5+i \\ 1-3i \end{bmatrix}$$

$$\implies \boxed{\begin{aligned} x &= \tfrac{5}{2} + \tfrac{1}{2}i \\ y &= \tfrac{1}{2} - \tfrac{3}{2}i \end{aligned}}$$

/5 **Problem 2:** Find all the (complex-valued) roots of $z^4 = 16$.

$$z^4 = 16e^{i(0+n2\pi)} \quad (n = 0, 1, 2, \ldots) \implies z = (16e^{i(n2\pi)})^{1/4} = 2e^{in\frac{\pi}{2}}$$

$$\begin{aligned} n = 0: &\quad z = 2e^{i0} = 2 \\ n = 1: &\quad z = 2e^{i\frac{\pi}{2}} = 2i \\ n = 2: &\quad z = 2e^{i\pi} = -2 \\ n = 3: &\quad z = 2e^{i\frac{3\pi}{2}} = -2i \end{aligned}$$

$$\implies \boxed{z = \pm 2, \pm 2i}$$