MATH 1300: Quiz #6 – SOLUTIONS

/6 **Problem 1:** Which of the following sets of vectors are linearly independent? Justify your answers.

(a) $\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}3\\4\end{bmatrix}$

/2

$$c_1\begin{bmatrix}1\\2\end{bmatrix} + c_2\begin{bmatrix}3\\4\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix} \Longrightarrow \begin{bmatrix}1 & 3 & 0\\2 & 4 & 0\end{bmatrix} \xrightarrow{\text{REF}} \begin{bmatrix}1 & 3 & 0\\0 & -2 & 0\end{bmatrix}$$

has only the trivial solution ($c_1 = c_2 = 0$) so YES, these vectors are linearly independent.

(b) $\begin{bmatrix}-1\\2\end{bmatrix}, \begin{bmatrix}3\\-6\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}$

/2

Any linearly independent set in $\mathbb{R}^2$ contains at most 2 vectors, so NO, these are *not* linearly independent.

(c) $\begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}-1\\1\\0\end{bmatrix}$

/2

$$c_1\begin{bmatrix}0\\1\\1\end{bmatrix} + c_2\begin{bmatrix}1\\0\\1\end{bmatrix} + c_3\begin{bmatrix}-1\\1\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} \Longrightarrow \begin{bmatrix}0 & 1 & -1 & 0\\1 & 0 & 1 & 0\\1 & 1 & 0 & 0\end{bmatrix} \xrightarrow{\text{REF}} \begin{bmatrix}1 & 0 & 1 & 0\\0 & 1 & -1 & 0\\0 & 0 & 0 & 0\end{bmatrix}$$

has infinitely many (non-trivial) solutions, so NO, these vectors are *not* linearly independent.

/4 **Problem 2:** Show that $\mathcal{B} = \left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}3\\4\end{bmatrix}\right\}$ is a basis for $\mathbb{R}^2$. Find the coordinates of $\mathbf{x} = \begin{bmatrix}1\\0\end{bmatrix}$ relative to $\mathcal{B}$.

From 1(a) we know the vectors in $\mathcal{B}$ are linearly independent. Since there are two of them, they span $\mathbb{R}^2$ and so form a basis for $\mathbb{R}^2$.

$$\mathbf{x}_{\mathcal{B}} = P_{\mathcal{B}}^{-1}\mathbf{x} = \begin{bmatrix}1 & 3\\2 & 4\end{bmatrix}^{-1}\begin{bmatrix}1\\0\end{bmatrix} = \frac{1}{-2}\begin{bmatrix}4 & -3\\-2 & 1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = -\frac{1}{2}\begin{bmatrix}4\\-2\end{bmatrix} = \begin{bmatrix}-2\\1\end{bmatrix}$$

check: $(-2)\begin{bmatrix}1\\2\end{bmatrix} + (1)\begin{bmatrix}3\\4\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix} = \mathbf{x} \quad \checkmark$