MATH 1300: Quiz #5 – SOLUTIONS

$/5$ **Problem 1:** Show that $\begin{bmatrix} a & 0 & d & c \\ b & 0 & -c & d \\ 0 & c & -b & a \\ 0 & d & a & b \end{bmatrix}$ is non-invertible for all values of the constants a, b, c, d.

$$\begin{aligned}
\begin{vmatrix} a & 0 & d & c \\ b & 0 & -c & d \\ 0 & c & -b & a \\ 0 & d & a & b \end{vmatrix} &= a \begin{vmatrix} 0 & -c & d \\ c & -b & a \\ d & a & b \end{vmatrix} - b \begin{vmatrix} 0 & d & c \\ c & -b & a \\ d & a & b \end{vmatrix} \\
&= a[-c(-bc - ad) + d(-ac + bd)] - b[-c(bd - ac) + d(ad + bc)] \\
&= ac(bc + ad) + ad(bd - ac) + bc(bd - ac) - bd(ad + bc) \\
&= (ad + bc)(ac - bd) + (ad + bc)(bd - ac) \\
&= (ad + bc)(ac - bd) - (ad + bd)(ac - bd) \\
&= 0
\end{aligned}$$

As its determinant is always 0, the given matrix is not invertible.

$/5$ **Problem 2:** Let $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$. Is $\mathbf{v} \in \text{span}\{\mathbf{u}_1, \mathbf{u}_2\}$?

Test whether $\mathbf{v}$ can be expressed as a linear combination of $\mathbf{u}_1$, $\mathbf{u}_2$:

$$c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$$

This gives a linear system for c_1, c_2, with augmented matrix:

$$\begin{bmatrix} 1 & 0 & 3 \\ 1 & 1 & 1 \\ 0 & 1 & -2 \end{bmatrix} \xrightarrow{R_2 - R_1} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 1 & -2 \end{bmatrix} \xrightarrow{R_3 - R_2} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

This system has a (unique) solution, so yes, $\mathbf{v} \in \text{span}\{\mathbf{u}_1, \mathbf{u}_2\}$.

In fact we have $c_1 = 3$, $c_2 = -2$, so $\mathbf{v}$ can be expressed as the linear combination

$$\mathbf{v} = 3\mathbf{u}_1 - 2\mathbf{u}_2.$$