MATH 1300: Quiz #4 – SOLUTIONS

/3 **Problem 1:** Solve by finding the inverse of the coefficient matrix: $\begin{cases} 5x_1 + 4x_2 = 9 \\ 6x_1 + 4x_2 = 10 \end{cases}$

Writing the system in matrix form:

$$\begin{bmatrix} 5 & 4 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 9 \\ 10 \end{bmatrix} \implies \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 6 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 9 \\ 10 \end{bmatrix}$$

$$= \frac{1}{(5)(4) - (4)(6)} \begin{bmatrix} 4 & -4 \\ -6 & 5 \end{bmatrix} \begin{bmatrix} 9 \\ 10 \end{bmatrix} = -\frac{1}{4} \begin{bmatrix} -4 \\ -4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\implies \boxed{x_1 = x_2 = 1}$$

/4 **Problem 2:** Find the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$.

$$[\,A \mid I\,] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3 - R_1]{R_2 - 2R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_3 + 2R_2]{R_1 - 2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 9 & 5 & -2 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right] \xrightarrow[R_2 - 3R_3]{R_1 + 9R_3;\, -R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right] = [\,I \mid A^{-1}\,]$$

$$\implies \boxed{A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}}$$

/3 **Problem 3:** Solve for X (assume A, B, X are invertible matrices):

$$(AXB^{-1} - AB^{-1})^{-1} = I$$

$$(AXB^{-1} - AB^{-1})^{-1} = I \implies AXB^{-1} - AB^{-1} = I^{-1} = I$$

$$\implies XB^{-1} - B^{-1} = A^{-1}$$

$$\implies X - I = A^{-1}B$$

$$\implies \boxed{X = A^{-1}B + I}$$