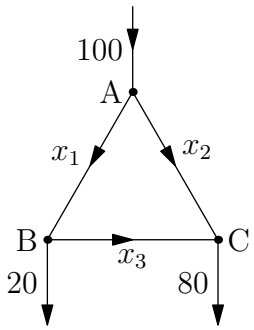
MATH 1300: Quiz #2 – SOLUTIONS

/5 **Problem 1:** The network shown represents (one-way) fluid flow in a system of pipes.



(a) Find the unknown flows in each branch.

balance each node: $\begin{cases} 100 = x_1 + x_2 \\ x_1 = 20 + x_3 \\ x_2 + x_3 = 80 \end{cases} \implies \begin{bmatrix} 1 & 1 & 0 & 100 \\ 1 & 0 & -1 & 20 \\ 0 & 1 & 1 & 80 \end{bmatrix}$

$$\xrightarrow{R_2 - R_1} \begin{bmatrix} 1 & 1 & 0 & 100 \\ 0 & -1 & -1 & -80 \\ 0 & 1 & 1 & 80 \end{bmatrix} \xrightarrow[R_1+R_2;\ -R_2]{R_3+R_2} \begin{bmatrix} 1 & 0 & -1 & 20 \\ 0 & 1 & 1 & 80 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\implies \boxed{\begin{cases} x_1 = 20 + t \\ x_2 = 80 - t \\ x_3 = t,\ t \in \mathbb{R} \end{cases}}$$

(b) Find the minimum flow in branch AB.

One-way flows require:

$$\begin{cases} 20 + t \geq 0 \\ 80 - t \geq 0 \\ t \geq 0 \end{cases} \implies \begin{cases} t \geq -20 \\ t \leq 80 \\ t \geq 0 \end{cases} \implies 0 \leq t \leq 80 \implies 20 \leq x_1 \leq 100$$

So the minimum flow in branch AB is $\boxed{20.}$

/5 **Problem 2:** Consider the following linear system:

$$\begin{cases} x_1 + 2x_2 + 2x_3 = 1 \\ x_2 + px_3 = 1 \\ -x_1 + x_2 + px_3 = 5 \end{cases}$$

For what value(s) of p does this system have (a) no solution? (b) a unique solution? (c) infinitely many solutions?

$$\begin{bmatrix} 1 & 2 & 2 & 1 \\ 0 & 1 & p & 1 \\ -1 & 1 & p & 5 \end{bmatrix} \xrightarrow{R_3+R_1} \begin{bmatrix} 1 & 2 & 2 & 1 \\ 0 & 1 & p & 1 \\ 0 & 3 & p+2 & 6 \end{bmatrix} \xrightarrow{R_3-3R_2} \begin{bmatrix} 1 & 2 & 2 & 1 \\ 0 & 1 & p & 1 \\ 0 & 0 & 2-2p & 3 \end{bmatrix}$$

(a) $2 - 2p = 0 \implies \boxed{p = 1}$

(b) $2 - 2p \neq 0 \implies \boxed{p \neq 1,\ p \in \mathbb{R}}$

(c) no value of p will give infinitely many solutions.