MATH 1300: Quiz #1 – SOLUTIONS

/3 **Problem 1:** How many solutions does the following system have? $\begin{cases} x + 2y - 3z = 1 \\ 3x + 6y + z = 13 \\ 4x + 8y - 2z = 9 \end{cases}$

$$\begin{bmatrix} 1 & 2 & -3 & 1 \\ 3 & 6 & 1 & 13 \\ 4 & 8 & -2 & 9 \end{bmatrix} \xrightarrow[R_3 - 4R_1]{R_2 - 3R_1} \begin{bmatrix} 1 & 2 & -3 & 1 \\ 0 & 0 & 10 & 10 \\ 0 & 0 & 10 & 5 \end{bmatrix} \xrightarrow{R_3 - R_2} \begin{bmatrix} 1 & 2 & -3 & 1 \\ 0 & 0 & 10 & 10 \\ 0 & 0 & 0 & -5 \end{bmatrix} \quad \text{(Row Echelon Form)}$$

The system is inconsistent: is has zero solutions.

/7 **Problem 2:** Consider the following system of equations: $\begin{cases} x_1 + 4x_2 - 4x_3 + 4x_4 = 5 \\ 2x_1 - x_2 + x_3 - x_4 = 1 \\ x_1 + x_2 - x_3 + x_4 = 2 \end{cases}$

(a) Without solving the system, try to predict the number of solutions: zero, one or infinitely many. Explain your reasoning.

With 3 equations, any row echelon form will have at most 3 pivots. There are 4 variables, so we should expect at least one free variable, hence infinitely many solutions—unless the system is inconsistent; without doing any calculation we can't be sure.

(b) Solve the system using Gaussian or Gauss-Jordan elimination.

$$\begin{bmatrix} 1 & 4 & -4 & 4 & 5 \\ 2 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & 1 & 2 \end{bmatrix} \xrightarrow[R_3 - R_1]{R_2 - 2R_1} \begin{bmatrix} 1 & 4 & -4 & 4 & 5 \\ 0 & -9 & 9 & -9 & -9 \\ 0 & -3 & 3 & -3 & -3 \end{bmatrix} \xrightarrow[-\frac{1}{9}R_2]{R_3 - \frac{1}{3}R_2} \begin{bmatrix} 1 & 4 & -4 & 4 & 5 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{(REF)}$$

$$\xrightarrow{R_1 - 4R_2} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{(RREF)}$$

$$\implies \begin{cases} x_1 = 1 \\ x_2 = 1 + x_3 - x_4 \\ x_3, x_4 \text{ are free.} \end{cases} \quad \text{or equivalently,} \quad \begin{cases} x_1 = 1 \\ x_2 = 1 + s - t \\ x_3 = s \in \mathbb{R} \\ x_4 = t \in \mathbb{R}. \end{cases}$$