Name: ______________________ Student #: ______________

THOMPSON RIVERS UNIVERSITY

MATH 1300
Linear Algebra for Engineers

Instructor: Richard Taylor

FINAL EXAM

5 December 2011 09:00–12:00

PROBLEM	GRADE	OUT OF
1		5
2		6
3		5
4		4
5		10
6		10
7		5
8		5
9		8
10		5
11		9
TOTAL:		72

Instructions:

1. Read all instructions carefully.
2. Read the whole exam before beginning.
3. Make sure you have all 9 pages.
4. Organization and neatness count.
5. You must clearly show your work to receive full credit.
6. You may use the backs of pages for calculations.
7. You may use an approved calculator.

/5

Problem 1: Find all solutions of the following system of equations:

$$\begin{aligned} x_1 + 2x_2 + x_3 + 3x_4 &= 4 \\ 3x_1 + 6x_2 + 5x_3 + 10x_4 &= 0 \\ 5x_1 + 10x_2 + 7x_3 + 17x_4 &= 23 \end{aligned}$$

/6 **Problem 2:** Consider the following linear system in which $\alpha, \beta \in \mathbb{R}$ are arbitary parameters:

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 11 \\ x_1 + x_2 + x_3 &= 6 \\ 5x_1 - x_2 + \alpha x_3 &= \beta \end{aligned}$$

Find the value(s) of α and β for which this system has

(a) no solution.

/3

(b) a unique solution.

/1

(c) infinitely many solutions.

/1

(d) Without doing any further calculation, determine the value(s) of α such that the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 5 & -1 & \alpha \end{bmatrix}$$

is *not* invertible.

/1

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Problem 3: Solve the matrix equation $AXB = C$ for X, where

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 11 & 11 \\ 29 & 27 \end{bmatrix}.$$

/4

Problem 4: Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} a & b & x \\ c & d & y \\ 0 & 0 & 1 \end{bmatrix}$ where $a, b, c, d, x, y \in \mathbb{R}$ are arbitrary. Show that if A is invertible then B is also invertible.

/10 **Problem 5:** Let $A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 2 \\ -2 & -2 & 1 \end{bmatrix}$.

(a) Without attempting to evaluate A^{-1}, determine whether A is invertible.
/3

(a) Evaluate A^{-1}.
/4

(b) Use your answer to part (a) to solve the following linear system.
/3

$$\begin{aligned} x_1 - x_2 + 3x_3 &= 1 \\ 2x_1 + x_2 + 2x_3 &= 2 \\ -2x_1 - 2x_2 + x_3 &= 3 \end{aligned}$$

/10

Problem 6: Consider the vectors

$$\mathbf{v}_1 = (1, 2, 1, 0) \qquad \mathbf{v}_2 = (0, -1, 3, k) \qquad \mathbf{v}_3 = (1, 1, -k, -4).$$

/3

(a) For what value(s) of k are $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent?

/2

(b) For what value(s) of k is it true that $\text{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\} = \mathbb{R}^4$?

/2

(c) For what value(s) of k are $\mathbf{v}_2, \mathbf{v}_3$ orthogonal to each other?

/3

(d) Let $\mathbf{x} = (4, 6, -4, -8)$. For what value(s) of k is $\mathbf{x} \in \text{span}\{\mathbf{v}_1, \mathbf{v}_3\}$?

/5 **Problem 7:** Find an equation (in any form) for the plane passing through the points

$$(1, 2, 3), \quad (-1, 2, 0), \quad \text{and} \quad (2, -3, 4).$$

/5 **Problem 8:** Find the distance from the point $(2, 2, 2)$ to the plane whose equation is $x + 2y + 3z = 6$.

/8 **Problem 9:** In a certain city, it is known that each year 30% of the people living downtown move to the suburbs, while 20% of the people living in the suburbs move downtown.

(a) Use this information to define a Markov chain and find its transition matrix.

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(b) At the steady state, what is the ratio of the suburban to the downtown population?

/3

(c) Write (but do not evaluate) a matrix expression that gives the population of each region after 6 years, if initially there are 5 million people living downtown and 3 million in the suburbs.

/2

/5 **Problem 10:** Let A be a 2×2 matrtrix with eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 9$, and corresponding eigenvectors $\mathbf{v}_1 = (1, -1)$ and $\mathbf{v}_2 = (1, 1)$.

(a) Find A.

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(b) Write (but do not evaluate) an expression for A^{15}.

/2

/9 **Problem 11:** Consider the following system of differential equations for functions $x(t)$, $y(t)$:

$$\begin{cases} x' = x + 3y \\ y' = 3x + y \end{cases}$$

(a) Find the general solution of this system.

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(b) Find a particular solution with initial conditions $x(0) = 1$, $y(0) = -2$.

/3