Name: _____

Student #: _____

GRADE: /42

Okanagan University College
Salmon Arm Campus

MATH 221 – Introduction to Linear Algebra

MIDTERM EXAM #2

25 March 2004

Instructor: Richard Taylor

Instructions:

1. *Read all instructions carefully.*
2. *Read the whole exam before beginning*; make sure you have all 8 pages.
3. Organize and write your solutions neatly. If you run out of room, continue your solution on the back of the page.
4. Where appropriate, show your work and explain your solution method—a correct final answer alone is not sufficient to guarantee full credit. Part marks may be awarded even if you don't obtain the final answer.

Problem 1: For what value(s) of k is the following matrix invertible?

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$$A = \begin{bmatrix} k & -k & 3 \\ 0 & k+1 & 1 \\ k & -8 & k-1 \end{bmatrix}$$

Problem 2: Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

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(a) Find the characteristic polynomial of A.

(b) Find the eigenvalues of A.

(c) Find the eigenvectors of A.

(d) Find a formula for $A^n\mathbf{x}$ if $\mathbf{x} = (1, 1, 1)$.

Problem 3: Construct a nondiagonal 2×2 matrix with eigenvalues 2 and 5.

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Problem 4: Consider the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \qquad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ k \end{bmatrix}$$

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(a) What value must k have in order for the set $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ to form an orthonormal basis for $\mathbb{R}^3$?

(b) Let k have the value you found in part (a), and let $\mathbf{w} = (1, 2, 3)$. Find the coordinates of $\mathbf{w}$ relative to the basis $\mathcal{B}$. That is, find $[\mathbf{w}]_{\mathcal{B}}$.

Problem 5: Let $\mathbf{v} = (0, -1, 1, 0)$ and $\mathbf{x} = (2, 1, 5, 3)$. Find the orthogonal projection of $\mathbf{x}$ onto Span$\{\mathbf{v}\}$.

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Problem 6: Let $\mathbf{v_1} = (1, 2, 1)$, $\mathbf{v}_2 = (1, -1, 1)$. The set $W = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$ is a plane in $\mathbb{R}^3$. Find the distance from W to the point $\mathbf{x} = (4, -2, 3)$.

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Problem 7: Consider the matrix

$$A = \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}.$$

Find a formula for A^n.

Problem 8: The set of points $(x_1, x_2, x_3) \in \mathbb{R}^3$ that satisfy the equation

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$$3x_1 + 2x_2 - 5x_3 = 4$$

form a plane.

(a) Write the equation of the plane in normal form.

(b) A second plane consists of points that satisfy

$$x_1 - 6x_2 - 2x_3 = 0.$$

Find the set of points that lie on the intersection of the two planes. Write your solution in vector form.

(c) Find the angle between the two planes.