Name: ________________ Student #: ________ GRADE: /31

THOMPSON RIVERS UNIVERSITY

MATH 212
Linear Algebra I

Instructor: Richard Taylor

MIDTERM EXAM #1

17 February 2006 08:30–09:20

Instructions:

1. *Read all instructions carefully.*
2. *Read the whole exam before beginning.*
3. *Make sure you have all 6 pages.*
4. Organize and write your solutions neatly.
5. You may use the backs of pages for calculations if necessary.
6. You must clearly show your work to receive full credit.
7. You may use a calculator.

***** THIS EXAM CONTAINS SOLUTIONS *****

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Problem 1: For what value(s) of k, if any, does the system of equations

$$3x + 2y = 11$$
$$6x + ky = 21$$

have:

(a) a unique solution?

$\boxed{k \neq 4}$

(b) no solution?

$\boxed{k = 4}$

(c) infinitely many solutions?

No value of k gives an infinite number of solutions.

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Problem 2: The following are the end result of performing Gauss-Jordan Elimination on augmented matrices for systems of three equations in three unknowns. In each case state the number of solutions, and what the solutions are (if any).

(a) $$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

One solution: $\begin{cases} x = 1 \\ y = 6 \\ z = -2 \end{cases}$

(b) $$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 2 \\ 0 & 1 & 7 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Infinitely many solutions: $\begin{cases} x = 2 + 3z \\ y = 4 - 7z \\ z \text{ is free} \end{cases}$

(c) $$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 8 \end{array}\right]$$

No solutions.

/5 **Problem 3:** (a) Use the Gauss-Jordan method to find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

$$\boxed{A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}}$$

(b) Using your answer to (a), find the solution of the linear system

$$\begin{aligned} x + y &= a \\ x + z &= b \\ y + z &= c \end{aligned}$$

where a, b and c are constants.

$$\boxed{\begin{cases} x = (a + b - c)/2 \\ y = (a - b + c)/2 \\ z = (-a + b + c)/2 \end{cases}}$$

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Problem 4: Prove the following:

If A and B are invertible matrices then the matrix AB is invertible and its inverse is $B^{-1}A^{-1}$.

Show that $(AB)(B^{-1}A^{-1}) = I$ and $(B^{-1}A^{-1})(AB) = I$

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Problem 5: Budget Rent-a-Car has two lots in town: one in the North, one in the South. Each week 20% of the cars rented at the North lot are returned to the South lot; 5% of the cars rented in the South are returned to the North. When the distribution of cars reaches an equilbrium, what percentage of the total number of cars will be in each of the two lots?

20% in North, 80% in South

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Problem 6: For each of the following matrices: (i) calculate its determinant, and (ii) determine whether the matrix is invertible.

(a) $A = \begin{bmatrix} 5 & 2 & 2 \\ -1 & 1 & 2 \\ 3 & 0 & 0 \end{bmatrix}$.

det $A = 6$ so A is invertible.

(b) $B = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

det $A = 0$ so A is not invertible.