Name: ______________________

Student #: ______________________

GRADE: /29

Okanagan University College

Salmon Arm Campus

MATH 221 – Introduction to Linear Algebra

MIDTERM EXAM #1

2 March 2004

Instructor: Richard Taylor

Instructions:

1. *Read all instructions carefully.*
2. *Read the whole exam before beginning*; make sure you have all 6 pages.
3. Organize and write your solutions neatly. If you run out of room, continue your solution on the back of the page.
4. Where appropriate, show your work and explain your solution method—a correct final answer alone is not sufficient to guarantee full credit. Part marks may be awarded even if you don't obtain the final answer.

Problem 1: For what value(s) of k, if any, does the system of equations

/4

$$\begin{aligned} 3x + 2y &= 11 \\ 6x + ky &= 21 \end{aligned}$$

have:

(a) a unique solution?

(b) no solution?

(c) infinitely many solutions?

Problem 2: Solve the system

/3

$$\begin{aligned} 4x_1 + 6x_2 &= 6 \\ 5x_1 + 9x_2 &= 18, \end{aligned}$$

using a matrix inverse.

/3

Problem 3: Let

$$\mathbf{u} = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} 3 \\ 6 \\ k \end{bmatrix}.$$

For what value(s) of k is $\mathbf{w}$ in the plane spanned by $\mathbf{u}$ and $\mathbf{v}$?

/3

Problem 4: Are the vectors $(1, -1, 2)$, $(3, 0, 1)$, and $(5, -2, 5)$ linearly independent?

Problem 5: Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the tranformation of the xy-plane that effects a reflection across the line $y = -x$. Find the standard matrix for T.

/3

Problem 6: Find the inverse of the matrix

/3

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

Problem 7: Let

/5

$$\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

(a) Let $\mathbf{x} = (-3, -3)$, and let $\mathcal{B} = \{\mathbf{v}, \mathbf{w}\}$ be a basis for $\mathbb{R}^2$. Find the coordinates of $\mathbf{x}$ relative to $\mathcal{B}$. That is, find $[\mathbf{x}]_{\mathcal{B}}$.

(b) Give a description and sketch of $\text{Span}\{\mathbf{v}\}$.

(c) Give a description of $\text{Span}\{\mathbf{v}, \mathbf{w}\}$.

Problem 8: The null space of the matrix

/5

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & -9 \\ 2 & 5 & 2 \end{bmatrix}$$

is a subspace of $\mathbb{R}^3$.

(a) Find a basis for $\operatorname{Nul} A$.

(b) Give a geometric description and a sketch of $\operatorname{Nul} A$.