Name: _____ Student #: _____ | GRADE: /50

Okanagan University College
Department of Mathematics & Statistics

# MATH 221 – Introduction to Linear Algebra
# FINAL EXAM

22 April 2005 09:00–12:00

Salmon Arm Campus

Instructor: Richard Taylor

**Instructions:**

- *Read all instructions carefully.*
- *Read the whole exam before beginning.*
- *Make sure you have all 8 pages.*
- Organize and write your solutions neatly.
- You may use the backs of pages for calculations if necessary.
- You must clearly show your work to receive full credit.

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**Problem 1:** Under what conditions (i.e. for what values of $a$, $b$ and $c$) will the linear system

$$\begin{aligned} x + y &= 2 \\ y + z &= 2 \\ x + z &= 2 \\ ax + by + cz &= 0 \end{aligned}$$

have: (a) a unique solution?

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(b) no solution?

/1

(c) an infinite number of solutions?

/1

/6 **Problem 2:** Consider the matrices

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 2 \\ 8 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0 \\ 4 & -2 \end{bmatrix}.$$

(a) Compute the matrix products $AC$ and $BC$, and show that $AC = BC$.

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(b) In ordinary algebra, the equation $AC = BC$ would imply that $A = B$. But clearly $A \neq B$ here. So what's going on? What property would the matrix $C$ have to have in order for $AC = BC$ to imply that $A = B$?

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**Problem 3:** (a) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{bmatrix}$$

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(b) Use your answer to part (a) to solve the linear system

$$\begin{aligned} x+\ y+\ z &= \ k \\ 3x+5y+4z &= -1 \\ 3x+6y+5z &= \ 0 \end{aligned}$$

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/6 **Problem 4:** (a) Compute the determinant of the matrix

$$A = \begin{bmatrix} k & 1 & 1 \\ 2 & 3 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

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(b) For what value(s) of $k$ does $A$ have an inverse?

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/9 **Problem 5:** Let $W = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$, where $\mathbf{v}_1 = (2, 2, 1)$, $\mathbf{v}_2 = (-2, 1, 2)$ (note that $W$ is a plane).

/2 (a) Show that $\mathbf{v}_1 \perp \mathbf{v}_2$.

/2 (b) Show that $\mathbf{b} = (0, 0, 3)$ is not in $W$.

/5 (c) Find the closest point in $W$ to the point $\mathbf{b}$.

/9 **Problem 6:** For the matrix $A = \begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix}$,

(a) Find the eigenvalues of $A$.

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(b) Find the corresponding eigenvectors.

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(c) Find a formula for $A^n$.

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/7 **Problem 7:** Find the best-fit (i.e. least squares) straight line through the points $(0, 0)$, $(1, 2)$, $(2, 2)$.