Name: ____________________ Student #: __________ GRADE: /52

Capilano College
Department of Mathematics & Statistics

MATH 126 – Calculus II
MIDTERM EXAM #2

20 July 2005 10:30–12:20

North Vancouver Campus

Instructor: Richard Taylor

Instructions:

- *Read all instructions carefully.*
- *Read the whole exam before beginning.*
- *Make sure you have all 7 pages.*
- Organize and write your solutions neatly.
- You may use the backs of pages for calculations if necessary.
- You must clearly show your work to receive full credit.

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Problem 1: For each of the following sequences… Does the sequence converge to a limit? If yes, find the limit.

(a) $\left\{ \dfrac{n^2 - n}{n^2 + n} \right\}_{n=1}^{\infty}$

(b) $\left\{ \frac{1}{2}\sin(2), \frac{1}{3}\sin(3), \frac{1}{4}\sin(4), \dots \right\}$

(c) $\left\{ \sqrt{n^2 + n} - n \right\}_{n=0}^{\infty}$

(d) $\left\{ \sqrt[n]{n} \right\}_{n=0}^{\infty}$

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Problem 2: For each of the following series… Does the series converge? Why / why not?

(a) $1 - 1 + 1 - 1 + \cdots$

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{3^k}{4^{k+2}}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n - \frac{1}{n}}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{2^n}$

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Problem 3: For each of the following series... Does the series converge? Why / why not?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{5}{n^4}$

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{5}{4^k + 3}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1000}{\sqrt{k}\, 3^k}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\ln(n)}$

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Problem 4: (a) Find the general solution $y(x)$ of the differential equation $y' = e^{y-x}$.

(b) Find the particular solution that satisfies the "initial value" $y(0) = 1$.

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Problem 5: Find the orthogonal trajectories for the family of curves $y = x^3 + C$. Draw a sketch illustrating your result.

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Problem 6: The volume of Shuswap Lake is 2×10^{13} L. At a rough estimate, recreational boaters spill 10^3 kg of gasoline into the lake each year. Fresh water enters the lake at a rate of 10^{12} L/yr, and water drains from the lake at the same rate. The lake is well mixed at all times.

(a) Find an expression for the quantity $x(t)$ of gasoline (in kg) in the lake after t years, supposing that initially there is no gasoline in the lake.

(b) If this process continues unabated, what will be the eventual equilibrium quantity of gasoline (in kg) in the lake?

(c) If the gas spillage were totally eliminated, what would be the "half life" of the remaining quantity of gasoline in the lake?

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Problem 7: We have seen that one power series representation of $\tan^{-1}(x)$ is given by

$$\tan^{-1}(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}.$$

(a) Find a power series representing the function $f(x) = x^2 \tan^{-1}(x^3)$.

(b) Find the interval of convergence of your series in (a).

(c) Use your series from (a) to evaluate $\int_0^{1/3} x^2 \tan^{-1}(x^3)\, dx$ to 6 decimal places.