

MATH 1240: Quiz #7 – SOLUTIONS

/6 **Problem 1:** Find the functions $y(x)$ that satisfy the following:

(a) $y'(x) = y \cos x, y(0) = 3.$

$$\begin{aligned}\frac{dy}{dx} = y \cos x &\implies \int \frac{dy}{y} = \int \cos x \, dx \\ &\implies \ln |y| = \sin x + C \\ &\implies |y| = e^{\sin x} \underbrace{e^C}_A \implies y = Ae^{\sin x} \quad (A \in \mathbb{R})\end{aligned}$$

$$y(0) = 3 = Ae^{\sin 0} \implies A = 3 \implies \boxed{y = 3e^{\sin x}}$$

(b) $y' + 2y = -4$

$$\begin{aligned}\frac{dy}{dx} = -2y - 4 &\implies \int \frac{dy}{2y + 4} = \int -dt \\ &\implies \frac{1}{2} \ln |2y + 4| = -t + C \implies \ln |2y + 4| = -2t + 2C \\ &\implies |2y + 4| = e^{-2t} \underbrace{e^{2C}}_A \implies 2y + 4 = Ae^{-2t} \\ &\implies 2y = -4 + Ae^{-2t} \implies \boxed{y = -2 + Be^{-2t} \quad (B \in \mathbb{R})}\end{aligned}$$

/4 **Problem 2:** A pot of boiling soup (100°C) is put in a cellar with a temperature of 10°C . After 30 minutes the soup has cooled to 80°C . When will the temperature reach 30°C ?

From Newton's law of cooling we have

$$T(t) = T_{\text{room}} + (T_0 - T_{\text{room}})e^{-kt} = 10 + (100 - 10)e^{-kt} = 10 + 90e^{-kt}.$$

At $t = 30$:

$$T(30) = 80 = 10 + 90e^{-30k} \implies k = -\frac{\ln(7/9)}{30}$$

At the time when $T = 30$:

$$T(t) = 30 = 10 + 90e^{-kt} \implies t = -\frac{\ln(2/9)}{k} = \boxed{30 \frac{\ln(2/9)}{\ln(7/9)} \approx 180 \text{ min}}$$