Prairie Discrete Math Workshop

Thompson Rivers University

May 3 & 4, 2013

Conference Program

Conference Sponsors
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THOMPSON RIVERS UNIVERSITY

Welcome to PDMW 2013

Welcome to the 2013 Prairie Discrete Math Workshop and welcome to Thompson Rivers University. The PDMW is a nearly annual meeting for researchers in discrete mathematics from the Prairie Region, plus neighbouring provinces and states. It has been held in various forms since 1995. Since 2002 it has been held at the Universities of Regina (2002, 2011), Lethbridge (2003, 2006), Manitoba (2008, 2010), Winnipeg (2005), UBC-Okanagan (2009), and Calgary (2012). This year is the first time the workshop has been held at Thompson Rivers University. TRU is located on traditional Secwepemc (Shuswap) territory in Kamloops, BC.

The program consists of six invited speakers and six contributed talks. The presentation of Bill Sands will be a joint session of the PDMW and the 41st annual BC Secondary Schools Mathematics Contest held at TRU.

We wish you a productive workshop and an enjoyable stay in Kamloops.

Rick Brewster
Paul Ottaway

Invited Speakers

- **James Currie**
  University of Winnipeg
  Avoiding Three Consecutive Blocks of the Same Size and Same Sum
- **Daryl Funk**
  Simon Fraser University
  Rearranging edge sets of cycles in graphs
- **Donovan Hare**
  University of British Columbia Okanagan
  The Cluster Deletion Problem for Cographs
- **Kieka Mynhardt**
  University of Victoria
  Secure Paired Domination
- **Shahla Nasserasr**
  University of Regina
  Partial Orders on Totally Nonnegative Matrices
- **Bill Sands**
  University of Calgary
  When worlds collide: math research vs. math contests

Contributed Talks

- **Sophie Burrill**, SFU
  Using generating trees to construct Skolem sequences
- **Ross Churchley**, SFU
  Finding tree-transverse matchings using local implications
- **Samuel Johnson**, SFU
  Towards a combinatorial understanding of lattice path asymptotics
- **Steve Melczer**, SFU
  Classifying Lattice Walks in the Quarter Plane
- **Andrew Poelstra**, SFU
  On Double Arithmetic Progressions
- **Laura Teshima**, UVic
  Broadcasts and Multipackings of Graphs

Speakers, Titles, and Abstracts

Sophie Burrill, *Simon Fraser University*

Using generating trees to construct Skolem sequences

A *Skolem sequence* is a linear arrangement of the multiset $\{1, 1, 2, 2, ..., n, n\}$ such that if r appears in positions i and j, then $|i - j| = r$. We first translate the problem to a particular set of perfect matchings, and then apply the method of generating trees for open arc diagrams to generate exhaustively all Skolem sequences of a given size. Tracking arc length between pairs of vertices in an arc diagram is the central task. Although we do not surpass previously known enumerative results, this method drastically reduces the search space compared to previously known methods.

James Currie, *University of Winnipeg*

Avoiding Three Consecutive Blocks of the Same Size and Same Sum

We show that there exists an infinite word over the alphabet $\{0, 1, 3, 4\}$ containing no three consecutive blocks of the same size and the same sum. This answers an open problem of Pirillo and Varricchio from 1994. This is joint work with Julien Cassaigne, Luke Schaeer and Jerey Shallit.

Ross Churchley, *Simon Fraser University*

Finding tree-transverse matchings using local implications

A matching M is called *T-transverse* if $G - M$ does not contain T as a subgraph. Finding T-transverse matchings is generally NP-complete – even for (large) trees – but efficient algorithms are known for a few small cases. In this talk, we will explain how the known algorithms for finding P_4- and chair-transverse matchings can be viewed as applications of the same general framework: each algorithm finds the constraints placed on the problem by certain "local" structures to transform it into an easily-solved problem like 2SAT or Perfect Matching. We will discuss how this framework helps construct certificates for the absence of a given type of matching, but also the difficulties in applying it to larger tree-transverse matching problems.

Daryl Funk, *Simon Fraser University*

Rearranging edge sets of cycles in graphs

Call the edge set of a cycle in a graph a *circuit*. A classical theorem of Whitney characterises when two graphs on the same edge set have the same collection of circuits. Members of a collection of subsets of a ground set having cycle-like behaviour are also said to be circuits. A theorem of Tutte tells us when such an abstract collection of circuits in fact forms the set of circuits of a graph. A *biased graph* is a graph together with a list of distinguished cycles. The circuits of a biased graph are given by edge sets of a natural collection of specified subgraphs. Two natural questions: When can the circuits of a biased graph be realised by the set of circuits of a graph? When do two biased graphs on the same edge set have the same collection of circuits? We provide some answers.

Donovan Hare, *University of British Columbia Okanagan*

The Cluster Deletion Problem for Cographs

The *min-edge clique partition problem* asks to find a partition of the vertices of a graph into a set of cliques with the fewest edges between cliques. This is a known NP-complete problem and has been studied extensively in the scope of *fixed-parameter tractability* (FPT) where it is commonly known as the Cluster Deletion problem. Many of the recently-developed FPT algorithms rely on being able to solve Cluster Deletion in polynomial time on restricted graph structures.

In this talk, we prove new structural properties of cographs (graphs that contain no induced path on 4 vertices) which characterize how a largest clique interacts with the rest of the graph. These results imply a remarkably simple polynomial time algorithm for Cluster Deletion on cographs. In contrast, we observe that Cluster Deletion remains NP-hard on a hereditary graph class which is slightly larger than cographs.

This joint work with Yong Gao and James Nastos.

Samuel Johnson, *Simon Fraser University*

Towards a combinatorial understanding of lattice path asymptotics

Lattice paths restricted to different regions is currently a very active area of research and a typical problem is to count the number of walks with steps from a given set $\mathcal{S}$ restricted to lie in a given region R. Recently, for walks with nearest neighbour steps restricted to the first quadrant of $\mathbb{Z}^2$, first order asymptotic estimates have been found. Unfortunately the approach uses very complicated techniques and the intuition is lost. We give a combinatorial approach that gives a tight upper bound on the exponential growth for these walks, and offer evidence that it is applicable to a very large class of models.

Steve Melczer, *Simon Fraser University*

Classifying Lattice Walks in the Quarter Plane

The enumeration of walks in the quarter plane with small steps is an elegant problem in combinatorics. Recently, much work has been done attempting to classify the seventy nine non-equivalent walks of this type according to analytic properties of their generating functions. We discuss this classification, its usefulness, and outline a method of classifying the final three walks not yet proven by other means. In particular, our method gives a parametrization of the generating function which illuminates the nature of its singularities, proving that they are infinite in number and giving dominant term asymptotics.

Kieka Mynhardt, *University of Victoria*

Secure Paired Domination

By placing guards on some of the vertices of a graph and allowing guards to move along edges to neighbouring vertices, we can "protect" the graph against attacks on its vertices or edges. Various models of attacks, guard configurations and guard movements are possible. Here we consider a model called *secure paired domination*. A *paired dominating set* of a graph G is a dominating set S such that the subgraph induced by S has a perfect matching.

Deploy guards on the vertices of a paired dominating set $S \subseteq V(G)$, one guard per vertex. In response to a single attack on a vertex of G without a guard, one or two guards move to neighbouring vertices, one of which is the attacked vertex. If the configuration of guarded vertices is a paired dominating set of G after any attack, then S is a *secure paired dominating set* of G. The guards then return to their original positions on S before the next attack.

Although we require that the guarded vertices form a paired dominating set before and after an attack, the definition above does not stipulate how the new perfect matching is related to the original one, nor how two guards move in relation to each other. There are several possibilities, all of which are different, yielding different *secure paired domination numbers*, the smallest number of guards in each type of secure paired dominating set. We discuss common bounds and their extremal graphs for these parameters, as well as extensions to eternal and m-eternal paired domination.

Shahla Nasserasr, *University of Regina*

Partial Orders on Totally Nonnegative Matrices

A matrix is called *totally nonnegative (positive)* if all of the minors of any order are nonnegative (positive). Such matrices are denoted by TN (TP) and have been of interest to many researchers for nearly a century. For the set of TN matrices of the same order, several partial orders such as *Bruhat, checkerboard,* and *compound partial orders* have been studied. I intend to survey some of these partial orders in the context of TN matrices and will present a number of their properties and applications.

Andrew Poelstra, *Simon Fraser University*

On Double Arithmetic Progressions

Van der Waerden's Theorem states that every finite coloring of the natural numbers must contain arbitrarily long monochromatic arithmetic progressions (sets of the form $\{x, x + d, x + 2d, \ldots, x + (k-1)d\}$ for some k). By considering the elements $x_1^r, x_2^r, x_3^r, \ldots$ of each color r, we extend this idea to *double arithmetic progressions* (DAP's): monochromatic arithmetic progressions whose indices are also arithmetic progressions.

We ask whether every finite coloring of the natural numbers must contain arbitrarily long DAP's, show that this question relates to complexity of words on finite alphabets, and present computational evidence. We also introduce the general-purpose computational tool `RamseyScript`.

Bill Sands, *University of Calgary*

When worlds collide: math research vs. math contests

Mathematical research can sometimes suggest suitable problems for math contests. Even less often, problems in math contests can inspire mathematical research. We will see a couple of examples of each.

Laura Teshima, *University of Victoria*

Broadcasts and Multipackings of Graphs

A *dominating broadcast* on a graph $G = (V, E)$ is a function $f : V \to \{0, 1, \ldots \mathrm{diam}(G)\}$ such that $f(v)$ is no more than the eccentricity of v, for all $v \in V$, and such that each vertex is within distance $f(u)$ from a vertex u with $f(u) > 0$. The broadcast domination number of a graph G, denoted $\gamma_b(G)$, is the minimum cost of a dominating broadcast on G. We present a new dual property to broadcast domination; a set $M \subseteq V$ is a multipacking of a graph G if, for each $v \in V$ and each s such that $1 \leq s \leq \mathrm{diam}(G)$, v is within distance s of at most s vertices in M. The *multipacking number*, $\mathrm{mp}(G)$, is the maximum cardinality of a multipacking of G. We outline some preliminary results on multipackings, discuss their connections to 2-packings and broadcasting, and suggest some future research directions.

Schedule

Time	Friday, May 3, 2013	Saturday, May 4, 2012
8:00-9:30		Breakfast (provided)
9:30-10:00	Registration, coffee	James Currie
10:00-10:30	Daryl Funk	
10:30-11:00		Andrew Poelstra
11:00-11:30	Laura Teshima	Coffee Break
11:30-12:00	Lunch (no host)	Kieka Mynhardt
12:00-12:30		
12:30-13:00		Lunch (no host)
13:00-13:30	Bill Sands	
13:30-14:00		
14:00-14:30	Break	
14:30-15:00	Ross Churchley	Steve Melczer
15:00-15:30	Sophie Burrill	Samuel Johnson
15:30-16:00	Shahla Nasserasr	Donovan Hare
16:00-16:30		

- Registration and coffee breaks will take place in HL 210A (House of Learning)
- All talks will take place in HL 204 with the exception of **Bill Sands** whose talk will be in the Barber Centre, HL 190.
- Breakfast on Saturday morning will be on the 4th floor of House of Learning