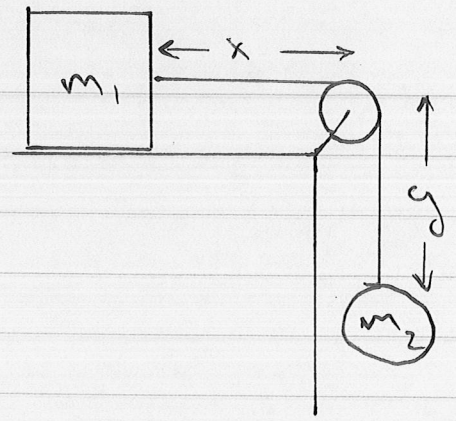
7.50



$$T = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2\dot{y}^2$$

$$U = -m_2 g y$$

$$\rightarrow L = T - U = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2\dot{y}^2 + m_2 g y$$

constraint: $\underbrace{x + y}_{f(x,y)} = l$

Equations of motion via Lagrange multipliers:

$$\frac{\partial L}{\partial x} + \lambda\frac{\partial f}{\partial x} = \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} \rightarrow \lambda = m_1\ddot{x} \qquad (1)$$

↖ tension force $= -\lambda$

$$\frac{\partial L}{\partial y} + \lambda\frac{\partial f}{\partial y} = \frac{d}{dt}\frac{\partial L}{\partial \dot{y}} \rightarrow m_2 g + \lambda = m_2\ddot{y} \qquad (2)$$

$$\ddot{x} + \ddot{y} = 0 \qquad (3)$$

$$(3) \rightarrow \frac{\lambda}{m_1} + \left(g + \frac{\lambda}{m_2}\right) = 0$$

$$\rightarrow \left(\frac{1}{m_1} + \frac{1}{m_2}\right)\lambda = -g \rightarrow \boxed{\lambda = -\frac{m_1 m_2}{m_1 + m_2}g}$$

(1) → $\boxed{\ddot{x} = -\left(\frac{m_2}{m_1+m_2}\right) \cdot g}$

(2) → $\boxed{\ddot{y} = g - \frac{m_1}{m_1+m_2} g = \frac{m_2}{m_1+m_2} g}$