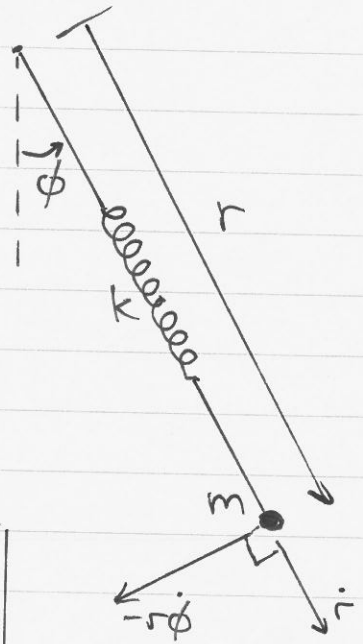
7.36




a) $T = \frac{1}{2}mv^2$

$= \frac{1}{2}m\left[\dot{r}^2 + (r\dot{\phi})^2\right]$

$= \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\phi}^2$

$U = \frac{1}{2}k(r-l_0)^2 - mgr\cos\phi$

→ $\boxed{L = T - U = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\phi}^2 - \frac{1}{2}k(r-l_0)^2 + mgr\cos\phi}$

b) Equations of motion:

$\frac{\partial L}{\partial r} = \frac{d}{dt}\frac{\partial L}{\partial \dot{r}}$ → $\boxed{mr\dot{\phi}^2 - k(r-l_0) + mg\cos\phi = m\ddot{r}}$ (1)

"centrifugal force" — spring force — radial component of gravity

$\frac{\partial L}{\partial \phi} = \frac{d}{dt}\frac{\partial L}{\partial \dot{\phi}}$ → $-mgr\sin\phi = mr^2\ddot{\phi} + 2mr\dot{r}\dot{\phi}$ → $\boxed{\ddot{\phi} = -\frac{g}{r}\sin\phi - 2\frac{\dot{r}}{r}\dot{\phi}}$ (2)

tangential torque due to gravity — Coriolis "force"

c) small oscillations: (equilibrium length: $\ddot{r} = \phi = \dot{\phi} = 0$

(1) $\Rightarrow r = l_0 + mg/k$ )

$r = l + \varepsilon \rightarrow \ddot{r} = \ddot{\varepsilon}$ ; $\quad \equiv l$

$\sin\phi = \phi - \frac{\phi^3}{3!} + \cdots \approx \phi$

$\cos\phi = 1 - \frac{\phi^2}{2} + \cdots \approx 1$

$\dot{\phi}^2 \approx 0$ (quadratic terms are negligible)

(1) $\rightarrow$ $-k\varepsilon$ ~~$-mg$~~ $= m\ddot{\varepsilon} \rightarrow$ $\boxed{\ddot{\varepsilon} + \frac{k}{m}\varepsilon = \cancel{0}}$ (*)

(2) $\rightarrow$ $\ddot{\phi} = -\frac{g}{l+\varepsilon}\sin\phi$ $\quad \left[\frac{1}{l+\varepsilon} = \frac{1}{l} - \frac{1}{l^2}\varepsilon + \cdots\right]$

$-2r\dot{r}\dot{\phi}$ ← negligible

$= -g\left(\frac{1}{l} - \frac{1}{l^2}\varepsilon + \cdots\right)(\phi - \cdots)$

$= -\frac{g}{l}\phi + \frac{g}{l^2}\varepsilon\phi + \cdots$

$\varepsilon\phi$: negligible

$\rightarrow$ $\boxed{\ddot{\phi} + \frac{g}{l}\phi = 0}$ (**)

(*) $\rightarrow$ $\varepsilon(t) = \frac{mg}{k} + Ae^{i\omega_1 t}$ with $\omega_1 = \sqrt{\frac{k}{m}}$ (osc. about eqm $\varepsilon_0 = \frac{mg}{k}$)

(**) $\rightarrow$ $\phi(t) = Be^{i\omega_2 t}$ with $\omega_2 = \sqrt{\frac{g}{l}}$