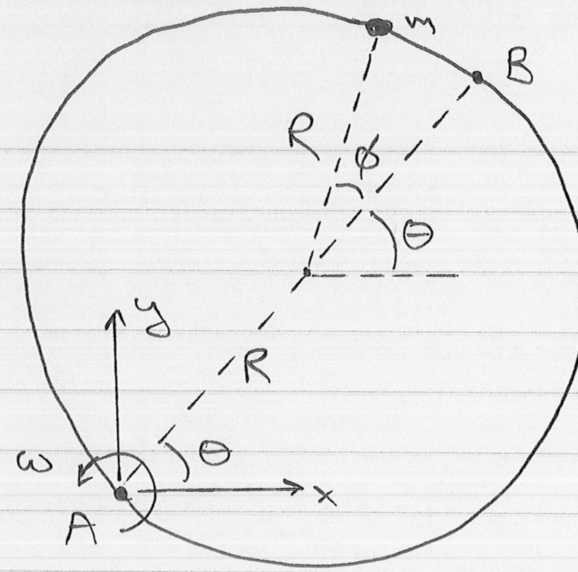

7.35

$$x = R\cos\theta + R\cos(\theta+\phi)$$
$$y = R\sin\theta + R\sin(\theta+\phi)$$

$$\rightarrow \dot{x} = -R\sin\theta\cdot\dot{\theta} - R\sin(\theta+\phi)(\dot{\theta}+\dot{\phi})$$
$$\dot{y} = R\cos\theta\cdot\dot{\theta} + R\cos(\theta+\phi)(\dot{\theta}+\dot{\phi})$$

$$\rightarrow \dot{x} = -\omega R\sin\theta - R(\omega+\dot{\phi})\sin(\theta+\phi)$$
$$\dot{y} = \omega R\cos\theta + R(\omega+\dot{\phi})\cos(\theta+\phi)$$

$$\rightarrow v^2 = \dot{x}^2 + \dot{y}^2$$

$$= \omega^2R^2\sin^2\theta + 2\omega R^2(\omega+\dot{\phi})\sin\theta\sin(\theta+\phi) + R^2(\omega+\dot{\phi})^2\sin^2(\theta+\phi)$$
$$+ \omega^2R^2\cos^2\theta + 2\omega R^2(\omega+\dot{\phi})\cos\theta\cos(\theta+\phi) + R^2(\omega+\dot{\phi})^2\cos^2(\theta+\phi)$$

$$= \omega^2R^2 + 2\omega R^2(\omega+\dot{\phi})\underbrace{\left[\sin\theta\sin(\theta+\phi)+\cos\theta\cos(\theta+\phi)\right]}_{\cos(\theta+\phi-\theta)=\cos\phi} + R^2(\omega+\dot{\phi})^2$$

$$= \omega^2R^2 + 2\omega R^2(\omega+\dot{\phi})\cos\phi + R^2(\omega+\dot{\phi})^2$$

So $L = T = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2R^2 + m\omega R^2(\omega+\dot{\phi})\cos\phi + \frac{1}{2}mR^2(\omega+\dot{\phi})^2$

Equation of motion:

$$\frac{\partial L}{\partial \phi} = \frac{d}{dt}\frac{\partial L}{\partial \dot{\phi}} \rightarrow -m\omega R^2(\omega+\dot{\phi})\sin\phi = \frac{d}{dt}\left[m\omega R^2\cos\phi + mR^2(\omega+\dot{\phi})\right]$$
$$= -m\omega R^2\sin\phi\cdot\dot{\phi} + mR^2\ddot{\phi}$$

$$\rightarrow \quad -m\omega^2 R^2 \sin\phi = mR^2 \ddot{\phi}$$

$$\rightarrow \quad \boxed{\ddot{\phi} + \omega^2 \sin\phi = 0}$$ ... same as simple pendulum with $\omega^2 = \frac{l}{g}$

Small oscillations: $\sin\phi \approx \phi$

$$\Rightarrow \ddot{\phi} + \omega^2 \phi = 0$$

assume $\phi(t) = e^{rt} \rightarrow r^2 + \omega^2 = 0 \rightarrow r = \pm i\omega$

$\Rightarrow$ frequency of small oscillations is ω.