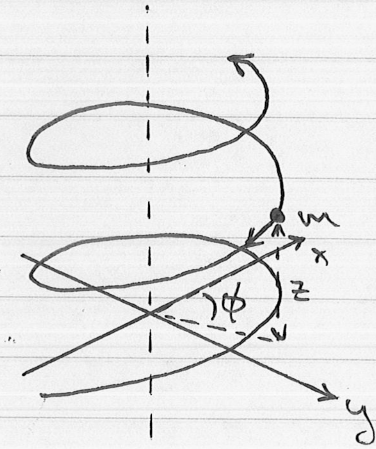
7.20

$$\begin{cases} z = \lambda\phi \\ x = R\cos\phi \\ y = R\sin\phi \end{cases} \longrightarrow \begin{cases} \dot{z} = \lambda\dot{\phi} \\ \dot{x} = -R\sin\phi\cdot\dot{\phi} \\ \dot{y} = R\cos\phi\cdot\dot{\phi} \end{cases}$$



$$T = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right)$$

$$= \frac{1}{2}m\left(R^2\sin^2\phi\cdot\dot{\phi}^2 + R^2\cos^2\phi\cdot\dot{\phi}^2 + \lambda^2\dot{\phi}^2\right)$$

$$= \frac{1}{2}m\left(R^2+\lambda^2\right)\dot{\phi}^2$$

$$= \frac{1}{2}m\left(R^2+\lambda^2\right)\left(\frac{\dot{z}}{\lambda}\right)^2 = \frac{1}{2}m\,\frac{R^2+\lambda^2}{\lambda^2}\cdot\dot{z}^2$$

$$U = -mgz$$

$$\rightarrow L = T - U = \frac{1}{2}m\,\frac{R^2+\lambda^2}{\lambda^2}\cdot\dot{z}^2 + mgz$$

Equation of motion:

$$\frac{\partial L}{\partial z} = \frac{d}{dt}\frac{\partial L}{\partial \dot{z}} \longrightarrow mg = \frac{d}{dt}\, m\,\frac{R^2+\lambda^2}{\lambda^2}\cdot\dot{z}$$

$$= m\,\frac{R^2+\lambda^2}{\lambda^2}\cdot\ddot{z}$$

$$\rightarrow \boxed{\ddot{z} = \frac{\lambda^2}{\lambda^2+R^2}\cdot g}$$

Note: as $R\to 0$ (vertical wire) we have $\ddot{z}\to g$ which makes sense (m is in free fall).