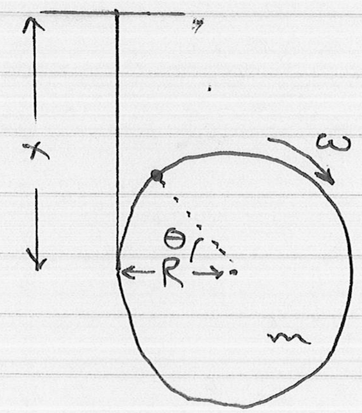
7.14



Uniform cylinder: $I = \frac{1}{2}mR^2$

$\rightarrow T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\omega^2$

but $dx = R\,d\theta \rightarrow \dot{x} = R\dot{\theta} = R\omega$ so:

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{\dot{x}}{R}\right)^2 = \frac{1}{2}m\dot{x}^2 + \frac{1}{4}m\dot{x}^2 = \frac{3}{4}m\dot{x}^2$$

$$U = -mgx$$

$$\Rightarrow L = T - U = \frac{3}{4}m\dot{x}^2 + mgx$$

Equation of motion:

$$\frac{\partial L}{\partial x} = \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} \longrightarrow mg = \frac{d}{dt}\,\frac{3}{2}m\dot{x} = \frac{3}{2}m\ddot{x}$$

$$\rightarrow \boxed{\ddot{x} = \frac{2}{3}g}$$