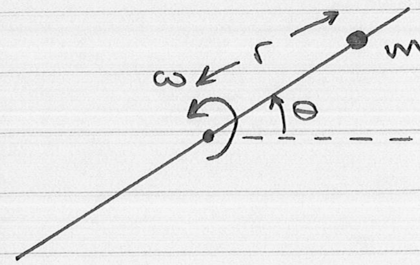
13.12 Mass m on frictionless rod spinning with const. ω in horizontal plane.



$$T = \frac{1}{2}m\left(\dot{r}^2 + (r\dot{\theta})^2\right)$$

$$= \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\omega^2$$

$$U = 0$$

$$\rightarrow L = T - U = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\omega^2$$

Generalized momentum:

$$p = \frac{\partial L}{\partial \dot{r}} = m\dot{r} \leftrightarrow \dot{r} = \frac{p}{m}$$

Hamiltonian:

$$H = p\dot{r} - L$$

$$= p\dot{r} - \underbrace{\frac{1}{2}m\dot{r}^2}_{p\dot{r}} - \frac{1}{2}mr^2\omega^2$$

$$= \frac{1}{2}p\dot{r} - \frac{1}{2}mr^2\omega^2$$

$$= \frac{p^2}{2m} - \frac{1}{2}mr^2\omega^2$$

Notice that $H \neq T + U = \frac{p^2}{2m} + \frac{1}{2}mr^2\omega^2$

because r is not a "natural coordinate" (relationship to x, y coords. is time-dependent).