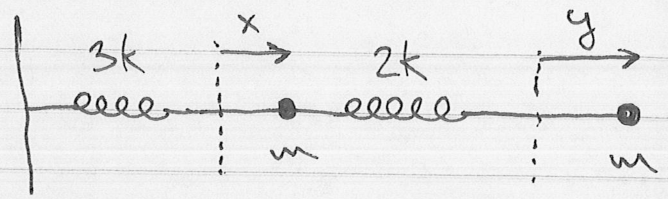
11.6 Same as 11.5 but with different springs.



Equations of motion:

$$\begin{aligned} m\ddot{x} &= -3kx + 2k(y-x) \\ m\ddot{y} &= -2k(y-x) \end{aligned} \Rightarrow \underbrace{\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix}}_{\ddot{\underline{x}}} = \underbrace{\begin{bmatrix} -5k & 2k \\ 2k & -2k \end{bmatrix}}_{K} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\underline{x}}$$

Assume $\underline{x}(t) = \underline{z}e^{i\omega t} \Rightarrow (K + \omega^2 M)\underline{z} = \underline{0}$

We have $K + \omega^2 M = \begin{bmatrix} \omega^2 m - 5k & 2k \\ 2k & \omega^2 m - 2k \end{bmatrix}$.

For non-trivial solutions we need:

$$\begin{aligned} 0 = \det(K + \omega^2 M) &= (\omega^2 m - 5k)(\omega^2 m - 2k) - 4k^2 \\ &= m^2\omega^4 - 7km\omega^2 + 6k^2 \end{aligned}$$

$$\begin{aligned} \Rightarrow \omega^2 &= \frac{7km \pm \sqrt{49k^2m^2 - 24k^2m^2}}{2m^2} \\ &= \frac{7 \pm 5}{2}\omega_0^2 \quad \text{with } \omega_0 = \sqrt{\frac{k}{m}} \\ &= \omega_0^2 \text{ or } 6\omega_0^2 \end{aligned}$$

$$\Rightarrow \boxed{\begin{aligned} \omega &= \omega_0 \text{ or } \sqrt{6}\,\omega_0 \\ &= \sqrt{\frac{k}{m}} \text{ or } \sqrt{\frac{6k}{m}} \end{aligned}}$$

Now solve for $\underline{z}$ to get modes...

1) case $\omega^2 = \omega_0^2 = \frac{k}{m}$:

$$\underline{0} = \left(K + \frac{k}{m} M\right)\underline{z} = \begin{bmatrix} -4k & 2k \\ 2k & -k \end{bmatrix} \underline{z} \xrightarrow{\text{RREF}} \left[\begin{array}{cc:c} -2 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

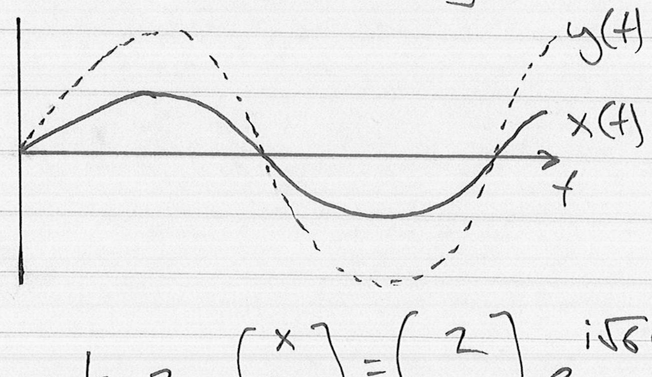
$$\Rightarrow \boxed{\underline{z} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}} \qquad \text{Motion is in-phase.}$$

2) case $\omega^2 = 6\omega_0^2 = 6\frac{k}{m}$:

$$\underline{0} = \left(K + 6\frac{k}{m} M\right)\underline{z} = \begin{bmatrix} k & 2k \\ 2k & 4k \end{bmatrix} \underline{z} \xrightarrow{\text{RREF}} \left[\begin{array}{cc:c} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$\Rightarrow \boxed{\underline{z} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}} \qquad \text{Motion is out-of-phase.}$$

Summary: mode 1, $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{i\omega_0 t}$



mode 2, $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{i\sqrt{6}\omega_0 t}$