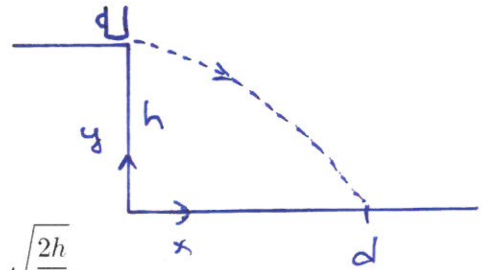
PHYS 1150: Quiz #5 – SOLUTIONS

/10 **Problem 1:** A customer in a restaurant slides an empty coffee mug down the counter. The height of the counter is h. The mug slides of the end of the counter and strikes the floor a distance d from the base of the counter.

(a) Derive (and simplify) a formula (in terms of h, d and the acceleration of gravity g) for the initial velocity at which the mug leaves the edge of the counter. (Express your answer using $\hat{\mathbf{\imath}}$, $\hat{\mathbf{\jmath}}$ unit vector notation.)

With the origin at the base of the counter, the position of the mug as a function of time is

$$\begin{aligned}\mathbf{x}(t) &= \mathbf{x}_0(t) + \mathbf{v}_0 t + \tfrac{1}{2}\mathbf{a}t^2 \\ &= h\hat{\mathbf{\jmath}} + v_0 t\hat{\mathbf{\imath}} + \tfrac{1}{2}(-g)t^2\hat{\mathbf{\jmath}} \\ &= (v_0 t)\hat{\mathbf{\imath}} + (h - \tfrac{1}{2}gt^2)\hat{\mathbf{\jmath}}.\end{aligned}$$



The mug strikes the floor when

$$0 = y(t) = h - \tfrac{1}{2}gt^2. \implies t^2 = \frac{2h}{g} \implies t = \sqrt{\frac{2h}{g}}.$$

(We can discard the negative root as being non-physical, since clearly the mug lands *after* it leaves the counter.) At the moment the mug strikes the floor we have

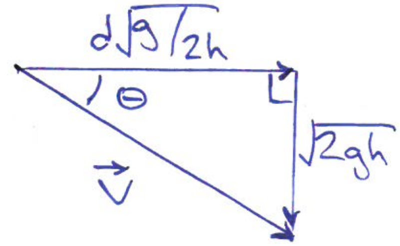
$$d = x(t) = v_0 t = v_0\sqrt{\frac{2h}{g}} \implies v_0 = d\sqrt{\frac{g}{2h}}$$

$$\implies \boxed{\mathbf{v}_0 = v_0\hat{\mathbf{\imath}} = d\sqrt{\frac{g}{2h}}\hat{\mathbf{\imath}}}$$

(b) Derive (and simplify) a formula for the mug's velocity as it strikes the floor. (Express your answer using $\hat{\mathbf{\imath}}$, $\hat{\mathbf{\jmath}}$ unit vector notation.)

The mug's velocity as a function of time is

$$\begin{aligned}\mathbf{v}(t) &= \mathbf{v_0} + \mathbf{a}t \\ &= v_0\hat{\mathbf{\imath}} - (gt)\hat{\mathbf{\jmath}}.\end{aligned}$$



At the moment the mug strikes the floor we have $t = \sqrt{\frac{2h}{g}}$ and so

$$\begin{aligned}\mathbf{v} &= d\sqrt{\frac{g}{2h}}\hat{\mathbf{\imath}} - \left(g\sqrt{\frac{2h}{g}}\right)\hat{\mathbf{\jmath}} \\ &\boxed{= d\sqrt{\frac{g}{2h}}\hat{\mathbf{\imath}} - \sqrt{2gh}\hat{\mathbf{\jmath}}.}\end{aligned}$$

The velocity's direction is therefore

$$\tan\theta = \frac{\sqrt{2gh}}{d\sqrt{\frac{g}{2h}}} = \frac{2h}{d} \implies \boxed{\theta = \operatorname{atan}\left(\frac{2h}{d}\right) \text{ (below horizontal)}}$$