Name: ______________________ Student #: ______________

THOMPSON RIVERS UNIVERSITY

PHYS 1150
Mechanics & Waves

Instructor: Richard Taylor

MIDTERM EXAM

SOLUTIONS

19 Oct 2018 13:30–14:20

Instructions:

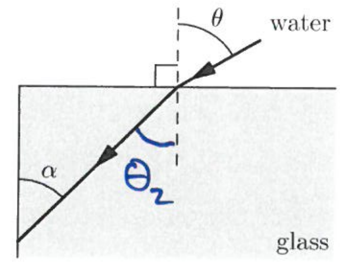
1. Read the whole exam before beginning.
2. Make sure you have all 5 pages.
3. Organization and neatness count.
4. Justify your answers.
5. Clearly show your work.
6. You may use the backs of pages for calculations.
7. You may use an approved formula sheet.
8. You may use an approved calculator.

PROBLEM	GRADE	OUT OF
1		7
2		8
3		8
4		6
TOTAL:		29

/7 **Problem 1:** A light ray is incident at an angle θ on the top surface of a block of glass (index of refraction $n = 1.52$) immersed in water ($n = 1.33$) as shown in the figure.

(a) Calculate the angle α for the case when $\theta = 45°$.

/3



$n_1 = 1.33$

$n_2 = 1.52$

Snell's law:

$$n_1 \sin\theta = n_2 \sin\theta_2$$

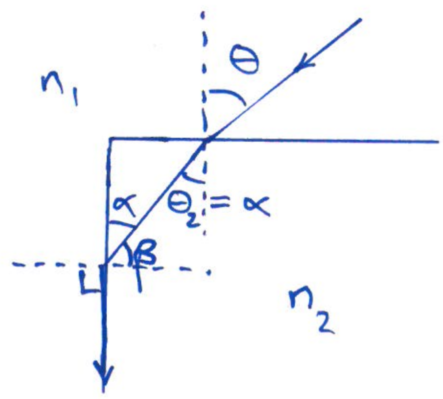
$$\Rightarrow (1.33)\sin 45^\circ = (1.52)\sin\theta_2$$

$$\Rightarrow \theta_2 = \sin^{-1}\left(\frac{1.33}{1.52}\sin 45^\circ\right) = 38.2^\circ$$

geometry gives $\alpha = \theta_2 = \boxed{38.2^\circ}$

(b) Find the maximum value of θ for which the refracted ray undergoes total internal reflection at the left face of the block.

/4



• refracted angle is 90°:

$$n_2 \sin\beta = n_1 \underbrace{\sin 90^\circ}_{1}$$

$$\Rightarrow (1.52)\sin\beta = 1.33$$

$$\Rightarrow \beta = \sin^{-1}\frac{1.33}{1.52} = 61.0^\circ$$

• geometry gives $\theta_2 = \alpha = 90^\circ - \beta$

$$= 90^\circ - 61.0^\circ = 29.0^\circ$$

• using Snell's law again:

$$n_1 \sin\theta = n_2 \sin\theta_2$$

$$\Rightarrow (1.33)\sin\theta = (1.52)\sin 29^\circ$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{1.52}{1.33}\sin 29^\circ\right) = \boxed{33.6^\circ}$$

(for larger θ the ray is refracted out into the water, and does not undergo total internal reflection)

/8 **Problem 2:** A concave spherical mirror has a radius of curvature of magnitude 24.0 cm. The mirror is used as a magnifier, so that when an object is placed in front of the mirror the resulting image is upright and larger than the object by a factor of 3.00.

(a) Calculate the object position.

/3

• upright image $\Rightarrow M = +3 = -\frac{q}{p} \Rightarrow q = -3p$

($q < 0$ so image is behind mirror)

• concave mirror: $f = \frac{R}{2} = \frac{24}{2} = 12.0\,\text{cm} > 0$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \Rightarrow \frac{1}{p} + \frac{1}{-3p} = \frac{1}{12}$$

$$\Rightarrow \frac{1}{p}\underbrace{\left(1 - \frac{1}{3}\right)}_{2/3} = \frac{1}{12}$$

$$\Rightarrow p = \frac{2}{3} \cdot 12 = \boxed{8.00\,\text{cm}}$$

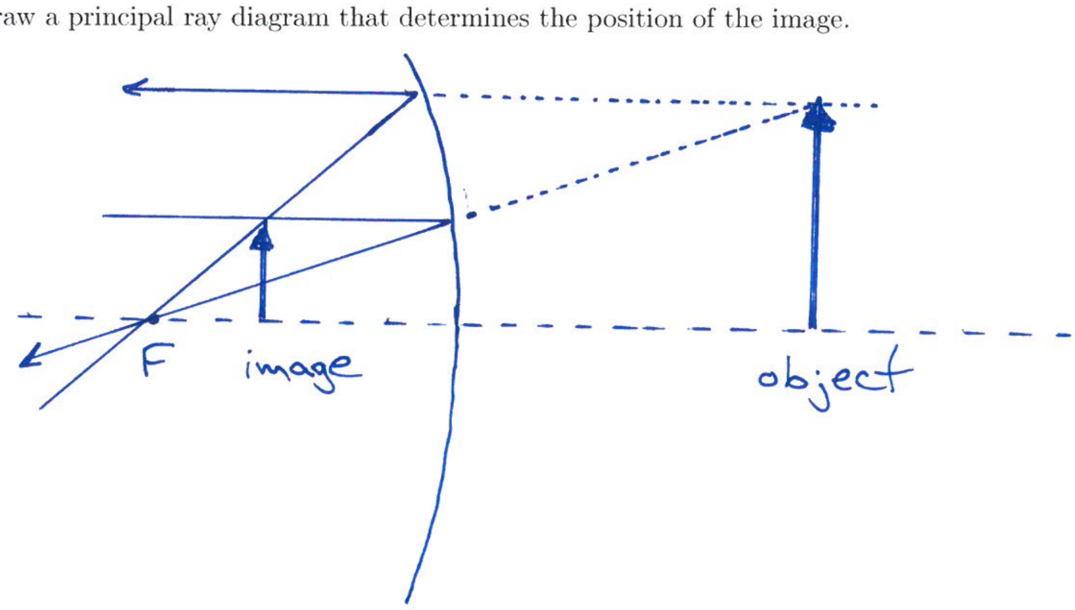
(b) Calculate the image position.

/1

$$q = -3p = -3(8) = \boxed{-24.0\,\text{cm (behind mirror)}}$$

(c) Draw a principal ray diagram that determines the position of the image.

/3



(d) Is the image real or virtual?

/1

virtual (since rays only appear to originate at image)

/8 **Problem 3:** A diverging lens has a focal length of magnitude 10.0 cm. An object is placed 15.0 cm from the lens.

(a) Calculate the image position.

/3

• diverging lens: $f = -10\,\text{cm} < 0$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}, \quad p = 15\,\text{cm}$$

$$\Rightarrow \frac{1}{15} + \frac{1}{q} = \frac{1}{-10}$$

$$\Rightarrow q = \left(-\frac{1}{10} - \frac{1}{15}\right)^{-1} = \boxed{-6.00\,\text{cm}}$$ (in front of lens, since $q < 0$)

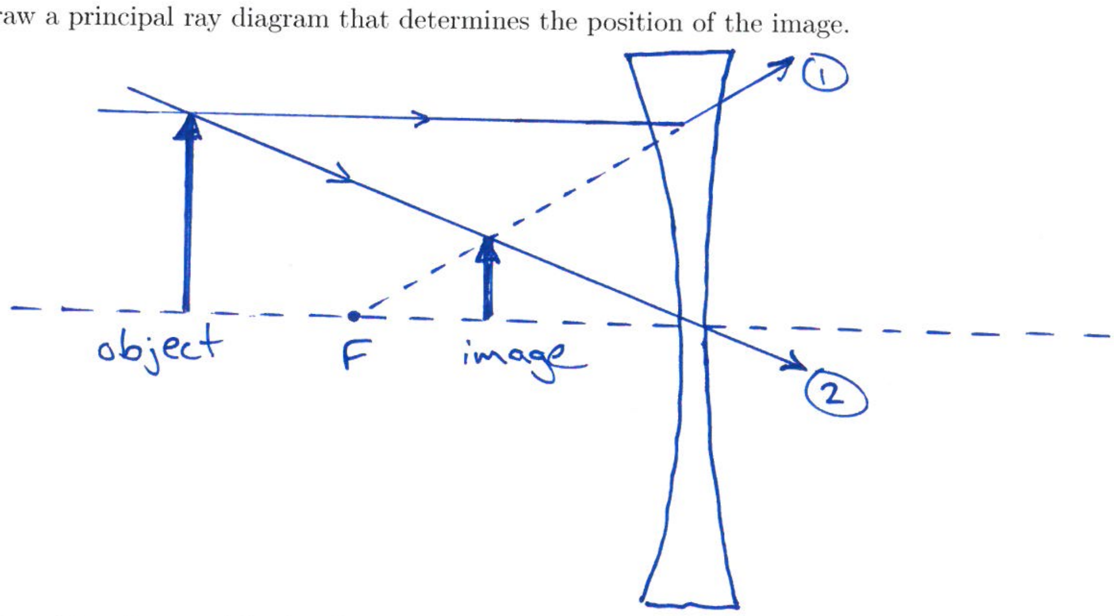
(b) Calculate the magnification.

/1

$$M = -\frac{q}{p} = -\frac{(-6)}{15} = \boxed{0.400}$$

(c) Draw a principal ray diagram that determines the position of the image.

/3



(d) If the object is moved farther from the lens, does the image get larger? or smaller?

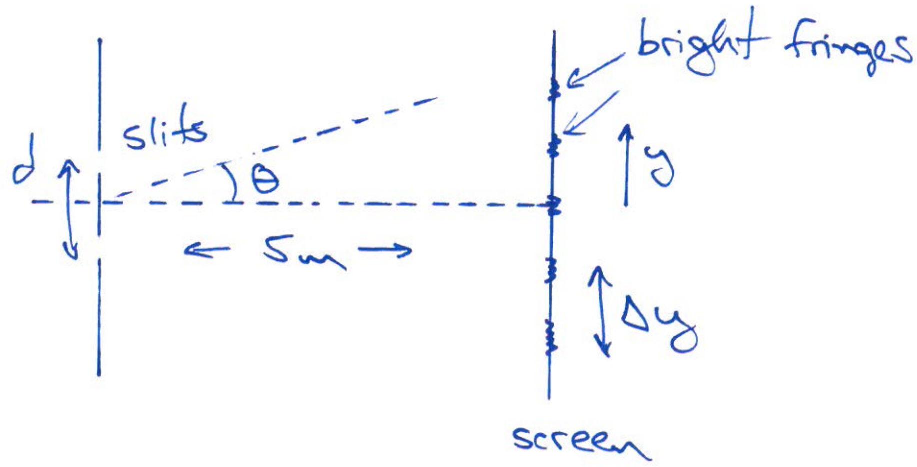
/1

smaller: ray 1 is unchanged but ray 2 is closer to horizontal ... image point moves toward F (and image gets smaller) as $p \to \infty$.

/6 **Problem 4:** A laser beam (wavelength 600 nm) is incident on two slits. A screen is placed 5.00 m from the slits and an interference pattern appears on the screen.

(a) What is the largest separation between the slits such that the bright fringes near the center of the interference pattern are separated by at least 2.00 mm?

/3



5m ≫ 2mm so can use small angle approx.:

bright fringes appear at $y \approx m\frac{\lambda L}{d}$ $(m = 0, \pm 1, \pm 2, \ldots)$

$\Rightarrow \Delta y = \frac{\lambda L}{d}$

$\Rightarrow d = \frac{\lambda L}{\Delta y} = \frac{(600\times 10^{-9})(5)}{2\times 10^{-3}} = 1.5\times 10^{-3}\,\text{m} = \boxed{1.50\,\text{mm}}$

(b) If the slit separation if 2.00 μm, how many bright bands appear on the screen?

/3

- bright bands occur where

$\sin\theta = m\frac{\lambda}{d} = m\cdot\frac{600\times 10^{-9}}{2\times 10^{-6}} = 0.3m$ $(m = 0, \pm 1, \pm 2, \ldots)$

- this is only possible if $-1 < 0.3m < 1$

$0.3m = 1 \Rightarrow m = \frac{1}{0.3} = 3.33$

$\therefore$ possible values of m are $\underbrace{0, \pm 1, \pm 2, \pm 3}$

$\boxed{\text{7 bright bands}}$