MATH 2650: Quiz #6 – SOLUTIONS

/5 **Problem 1:** Evaluate $\iint_D \frac{y^2}{x^2+y^2}\,dA$ where D is the region between the circles $x^2+y^2=a^2$ and $x^2+y^2=b^2$ with $0<a<b$.

Convert to polar coordinates ($y = r\sin\theta$, $x^2+y^2=r^2$, $dA = r\,dr\,d\theta$):

$$\int_0^{2\pi}\int_a^b \frac{(r\sin\theta)^2}{r^2}(r\,dr\,d\theta) = \int_0^{2\pi}\int_a^b r\sin^2\theta\,dr\,d\theta$$
$$= \underbrace{\int_0^{2\pi}\sin^2\theta\,d\theta}_{\pi}\cdot\underbrace{\int_a^b r\,dr}_{\frac{1}{2}r^2\Big|_a^b} = \boxed{\frac{\pi}{2}(b^2-a^2)}$$

/5 **Problem 2:** Evaluate $\int_0^a\int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}}(2x+y)\,dx\,dy$ by converting to polar coordinates.

$$= \iint_D (2x+y)\,dA$$

where D is the region within the circle $x^2+y^2=a^2$ above the x-axis. In polar coordinates ($x = r\cos\theta$, $y = r\sin\theta$, $dA = r\,dr\,d\theta$):

$$= \int_0^{\pi}\int_0^a (2r\cos\theta + r\sin\theta)(r\,dr\,d\theta)$$
$$= \int_0^{\pi}\int_0^a r^2(2\cos\theta+\sin\theta)\,dr\,d\theta$$
$$= \int_0^{\pi}(2\cos\theta+\sin\theta)\,d\theta\cdot\int_0^a r^2\,dr$$
$$= \Big[2\sin\theta-\cos\theta\Big]_0^{\pi}\cdot\Big[\tfrac{1}{3}r^2\Big]_0^a = 2\cdot\frac{a^3}{3} = \boxed{\frac{2}{3}a^3}$$