

MATH 2650: Quiz #5 – SOLUTIONS

/5 **Problem 1:** Evaluate $\iint_D y\sqrt{x^2 - y^2} dA$ where $D = \{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq x\}$.

$$\begin{aligned}
 \iint_D y\sqrt{x^2 - y^2} dA &= \int_0^2 \int_0^x y\sqrt{x^2 - y^2} dy dx \\
 &= \int_0^2 \int_{x^2}^0 u^{1/2} \left(-\frac{1}{2} du\right) dx \quad (u = x^2 - y^2, du = -2y dy) \\
 &= \int_0^2 \left[\frac{1}{3}u^{3/2}\right]_0^{x^2} dx \\
 &= \int_0^2 \frac{1}{3}x^3 dx = \frac{1}{12}x^4 \Big|_0^2 = \boxed{\frac{4}{3}}
 \end{aligned}$$

/5 **Problem 2:** Evaluate $\int_0^1 \int_{x^2}^1 \sqrt{y} \sin y dy dx$.

Change the order of integration:

$$\begin{aligned}
 \int_0^1 \int_{x^2}^1 \sqrt{y} \sin y dy dx &= \iint_D \sqrt{y} \sin y dA \quad \text{where } D = \{(x, y) : x^2 \leq y \leq 1, 0 \leq x \leq 1\} \\
 &= \{(x, y) : 0 \leq x \leq \sqrt{y}, 0 \leq y \leq 1\}. \\
 &= \int_0^1 \int_0^{\sqrt{y}} \sqrt{y} \sin y dx dy \\
 &= \int_0^1 \left[x\sqrt{y} \sin y \right]_{x=0}^{x=\sqrt{y}} dy \\
 &= \int_0^1 y \sin y dy \\
 &= -y \cos y \Big|_0^1 - \int_0^1 -\cos y dy \quad (u = y, dv = \sin y dy; du = dy, v = -\cos y) \\
 &= \sin y - y \cos y \Big|_0^1 \\
 &= \boxed{\sin 1 - \cos 1}
 \end{aligned}$$