MATH 2650: Quiz #4 – SOLUTIONS

/5 **Problem 1:** Consider the parametric curve given by the vector function $\mathbf{r}(t) = (t, 3\cos t, 3\sin t)$.

(a) Calculate the unit tangent vector $\mathbf{T}$ at the point where $t = \pi$.

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$$\mathbf{r}'(t) = (1, -3\sin t, 3\cos t) \implies \mathbf{r}'(\pi) = (1, 0, -3).$$

$$\mathbf{T}' = \frac{\mathbf{r}'}{|\mathbf{r}'|} = \frac{(1,0,-3)}{\sqrt{1^2 + 0^2 + (-3)^2}} = \boxed{\frac{1}{\sqrt{10}}(1,0,-3)}$$

(b) Calculate the curvature $\kappa(t)$.

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Using the results of the previous questions we get

$$\kappa(t) = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\frac{1}{\sqrt{10}}(0, -3\cos t, -3\sin t)|}{|(1, -3\sin t, 3\cos t)|} = \frac{\frac{1}{\sqrt{10}}\sqrt{0^2 + (-3\cos t)^2 + (-3\sin t)^2}}{\sqrt{1^2 + (-3\sin t)^2 + (3\cos t)^2}} = \frac{\frac{1}{\sqrt{10}} \cdot 3}{\sqrt{10}} = \boxed{\frac{3}{10}}$$

/5 **Problem 2:** Let C be the curve of intersection of the surfaces $x^2 = 2y$ and $3z = xy$. Find the exact length of C from the origin to the point $(6, 18, 36)$.

First represent the curve parametrically. Parameterizing with respect to $x = t$ gives

$$x(t) = t, \quad y = \tfrac{1}{2}x^2 = \tfrac{1}{2}t^2, \quad z = \tfrac{1}{3}xy = \tfrac{1}{3}(t)(\tfrac{1}{2}t^2) = \tfrac{1}{6}t^3.$$

The length of the curve is then

$$\begin{aligned}
L &= \int_0^6 \sqrt{x'^2 + y'^2 + z'^2}\, dt \\
&= \int_0^6 \sqrt{(1)^2 + (t)^2 + (\tfrac{1}{2}t^2)^2}\, dt \\
&= \int_0^6 \sqrt{1 + t^2 + \tfrac{1}{4}t^4}\, dt \\
&= \int_0^6 \sqrt{(1 + \tfrac{1}{2}t^2)^2}\, dt \\
&= \int_0^6 1 + \tfrac{1}{2}t^2\, dt = \left[t + \tfrac{1}{6}t^3\right]_0^6 = 6 + \tfrac{1}{6}6^3 = \boxed{42}
\end{aligned}$$