MATH 2650: Quiz #2 – SOLUTIONS

/4 **Problem 1:** Evaluate $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$ when $s = 1$, $t = 2$:

$$z = \sqrt{x}e^{xy}, \quad x = 1 + st, \quad y = s^2 - t^2.$$

$$\begin{matrix} s = 1 \\ t = 2 \end{matrix} \implies \begin{matrix} x = 3 \\ y = -3 \end{matrix} \implies \begin{matrix} \dfrac{\partial x}{\partial s} = t = 2, & \dfrac{\partial y}{\partial s} = 2s = 2 \\ \dfrac{\partial x}{\partial t} = s = 1, & \dfrac{\partial y}{\partial t} = -2t = -4 \end{matrix}$$

$$\frac{\partial z}{\partial x} = \tfrac{1}{2}x^{-1/2}e^{xy} + x^{1/2}e^{xy}y = \tfrac{1}{2}3^{-1/2}e^{-9} + 3^{1/2}e^{-9}(-3) = \left(\frac{1}{2\sqrt{3}} - 3\sqrt{3}\right)e^{-9}$$

$$\frac{\partial z}{\partial y} = \sqrt{x}e^{xy}x = 3\sqrt{3}e^{-9}$$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} = \left(\frac{1}{2\sqrt{3}} - 3\sqrt{3}\right)e^{-9}(2) + 3\sqrt{3}e^{-9}(2) = \boxed{\frac{1}{\sqrt{3}e^{9}}}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} = \left(\frac{1}{2\sqrt{3}} - 3\sqrt{3}\right)e^{-9}(1) + 3\sqrt{3}e^{-9}(-4) = \boxed{\left(\frac{1}{2\sqrt{3}} - 15\sqrt{3}\right)e^{-9}}$$

/6 **Problem 2:** Over a certain region of space the electric potential V [in Volts] is given by

$$V(x, y, z) = 5x^2 - 3xy + xyz$$

[with x, y, z in meters].

(a) Find the rate of change of V at $P(3, 4, 5)$ in the direction $\mathbf{v} = (1, 1, -1)$.

We have

$$\nabla V = (10x - 3y + yz, -3x + xz, xy) \implies \nabla V(3, 4, 5) = (38, 6, 12).$$

Now form the unit vector $\mathbf{u} = \mathbf{v}/|\mathbf{v}| = (1, 1, -1)/\sqrt{3}$. Then,

$$D_{\mathbf{u}}V = \nabla V \cdot \mathbf{u} = (38, 6, 12) \cdot (1, 1, -1)/\sqrt{3} = \boxed{32/\sqrt{3} \approx 18.5\,\mathrm{V/m}}$$

(b) In what direction does V increase most rapidly at P?

In the direction of $\nabla V(3, 4, 5) = (38, 6, 12)$.

(c) What is the maximum rate of increase of V at P?

$$|\nabla V(3, 4, 5)| = |(38, 6, 12)| = \sqrt{38^2 + 6^2 + 12^2} = \boxed{\sqrt{1624} \approx 40.3\,\mathrm{V/m}}$$