

Name: _____

GRADE: /31

Student #: _____

OKANAGAN UNIVERSITY COLLEGE
Salmon Arm Campus

MATH 221 – Introduction to Linear Algebra
MIDTERM EXAM #1

2 March 2005

Instructor: Richard Taylor

Time allowed: 80 minutes

Instructions:

1. *Read all instructions carefully.*
2. *Read the whole exam before beginning;* make sure you have all 6 pages.
3. Organize and write your solutions neatly. If you run out of room, continue your solution on the back of the page.
4. Where appropriate, show your work and explain your solution method—a correct final answer alone is not sufficient to guarantee full credit. Part marks may be awarded even if you don't obtain the final answer.

Problem 1: (a) Find the general solution, and one particular solution, of the linear system

/6

$$4x_1 - 2x_2 + 6x_3 = 0$$

$$-2x_1 + x_2 + 3x_3 = 1$$

(b) Write the general solution in vector form.

(c) What geometric interpretation can be given to the solution set?

Problem 2: Use a matrix inverse to solve the linear system

/4

$$2x + y = a$$

$$3x + 6y = b$$

where a and b are constants.

Problem 3: For which value(s) of a does the system of equations

/5

$$x + 2y - 3z = 4$$

$$3x - y + 5z = 2$$

$$4x + y + (a^2 - 14)z = a + 2$$

have:

(a) no solutions?

(b) infinitely many solutions?

(c) exactly one solution?

/6

Problem 4: (a) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 3 & 0 \\ 1 & 3 & 5 \end{bmatrix}$.

(b) How could you check your answer to (a) without using a calculator?

(c) Use your answer to (a) to solve the following linear system.

$$w = 6$$

$$w + 3x = 30$$

$$w + 3x + 5y = 35$$

Problem 5: Find the standard matrix for the composite linear transformation of \mathbb{R}^2 that effects a projection onto the x -axis, followed by a clockwise rotation by 60° , followed by a reflection across the y -axis.

/5

Problem 6: Canada is divided into three basic geographic regions: the Western provinces, the Eastern provinces, and the Northern territories. It is found that each year 5% of the residents in the West move to the North and 15% move to the East. Of the residents in the East, 10% move to the West and 5% move to the North. And of the residents in the North, 10% move to the West and 15% move to the East.

/5

If the migration patterns remain unchanged, what percentage of the population will reside in each the three regions after a long period of time?