MATH 1240: Quiz #1 – SOLUTIONS

/4 **Problem 1:** Use the definition of the definite integral (as a limit of Riemann sums) to evaluate $\int_0^2 (x^2+1)\,dx$.

With $f(x) = x^2+1$, $\Delta x = \frac{2}{n}$ and $x_i = 0 + i\Delta x = \frac{2i}{n}$:

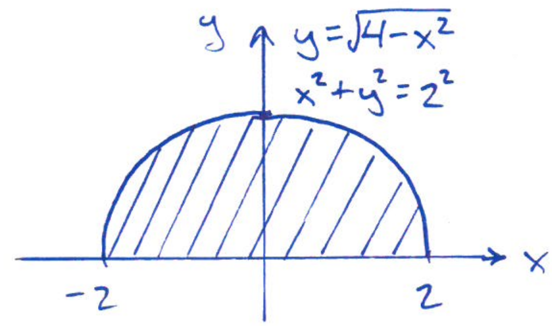
$$\begin{aligned}
\int_0^2 (x^2+1)\,dx &= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{2i}{n}\right)^2 + 1\right]\frac{2}{n} \\
&= \lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left[\frac{4i^2}{n^2} + 1\right] \\
&= \lim_{n\to\infty} \frac{2}{n} \left(\frac{4}{n^2}\sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} 1\right) \\
&= \lim_{n\to\infty} \frac{2}{n} \left(\frac{4}{n^2}\frac{n(n+1)(2n+1)}{6} + n\right) \\
&= \lim_{n\to\infty} \frac{8}{n^2}\frac{(n+1)(2n+1)}{6} + 2 \\
&= \lim_{n\to\infty} \frac{16n^2+\cdots}{6n^2} + 2 = \frac{8}{3} + 2 = \boxed{\frac{14}{3}}
\end{aligned}$$

/6 **Problem 2:** Use geometry (not Riemann sums) to evaluate:

/3 (a) $\int_{-2}^{2} \sqrt{4-x^2}\,dx$

area of half-circle:

$= \frac{1}{2}\cdot\pi(2^2) = \boxed{2\pi}$

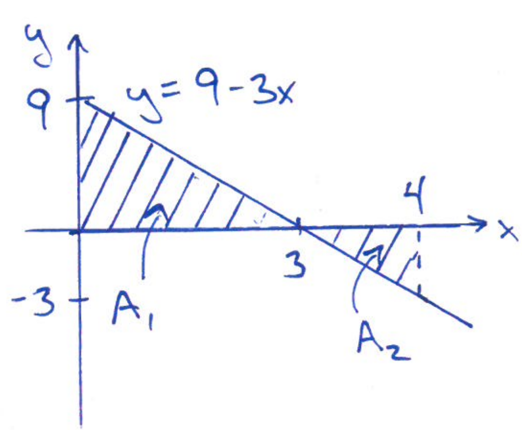


/3 (b) $\int_0^4 (9-3x)\,dx$

net area:

$A_1 - A_2 = \frac{1}{2}(3)(9) - \frac{1}{2}(1)(3)$

$= \frac{27-3}{2} = \boxed{12}$



16/01/2019