Name: _____ Student #: _____

THOMPSON RIVERS UNIVERSITY

MATH 115
Calculus for the Biological Sciences

Instructor: Richard Taylor

MIDTERM EXAM #1

14 October 2005 14:30–15:20

Instructions:

1. Read all instructions carefully.
2. Read the whole exam before beginning.
3. Make sure you have all 6 pages.
4. Organize and write your solutions neatly.
5. You may use the backs of pages for calculations.
6. You must clearly show your work to receive full credit.

PROBLEM	GRADE	OUT OF
1		3
2		5
3		9
4		3
5		5
6		5
7		5
TOTAL:		35

/3 **Problem 1:** Evaluate $\lim_{x \to 2} \dfrac{x^2 + 5x - 14}{x^2 - 4}$

/5 **Problem 2:** (a) State the definition of the derivative, $f'(x)$.

(b) Using your definition from part (a), find the derivative of the function

$$f(x) = 3x^2$$

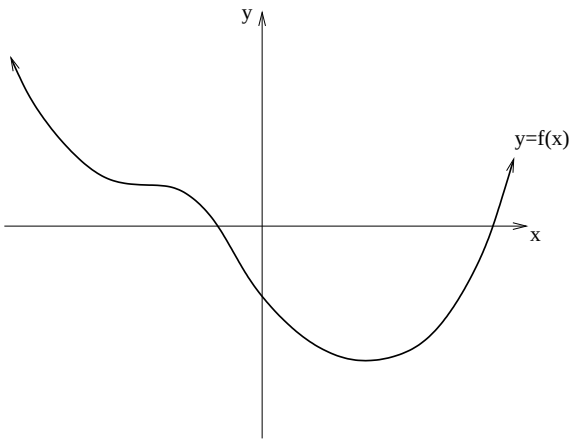
/9 **Problem 3:** Differentiate each of the following functions. *Do not simplify your answers.*

/3 (a) $f(x) = \sqrt{2 - 5x}$

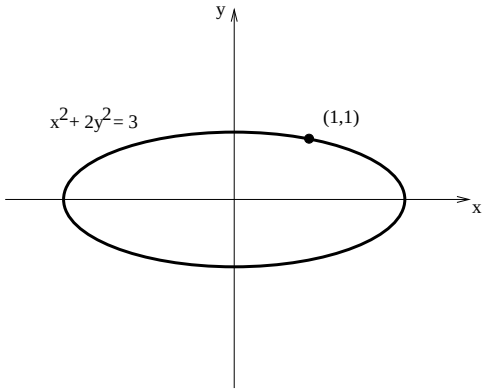
/3 (b) $h(x) = x^3(\ln x)^2$

/3 (c) $g(x) = \dfrac{e^{3x}}{\sin x + x^2}$

/3 **Problem 4:** On the same axes as the graph of $y = f(x)$ shown below, sketch the graph of $y = f'(x)$.



/5 **Problem 5:** The graph of the equation $x^2 + 2y^2 = 3$ is an ellipse. Use implicit differentiation to find the slope of the tangent line to this ellipse at the point $(1, 1)$.



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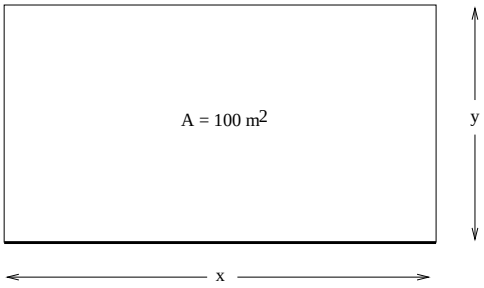
Problem 6: A farmer wants to fence a rectangular enclosure that must have an area of $100\,\mathrm{m}^2$. On three sides of the enclosure, she will use ordinary fencing that costs \$1 per meter. On the fourth side (the thicker line in the diagram below) she will to use titanium fencing that costs \$5 per meter. What should be the dimensions of the enclosure, in order to minimize the total cost of fencing material?



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Problem 7: The human heart pumps about $70\,\text{cm}^3$ of blood per beat[1]. So, during strenuous exercise when your pulse rate is 120 beats per minute, your heart fills with blood at each beat (and therefore increases its volume) at a rate of about $2 \times 70\,\text{cm}^3 \times 120\,\text{bpm}/60\,\text{sec} \approx 280\,\text{cm}^3/\text{sec}$.

Approximate the heart as a sphere, and suppose its diameter is about $8\,\text{cm}$. At what rate does the heart's diameter increase as it draws in blood with each beat?

(Note that the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$).

[1] Curtis, Helena. Biology: 5th Edition. New York: Worth, 1989: 756.