## MATH 114 - Review Problems

## A. Limits

1. Refer to the graph to the right.

Find: (a) $\lim _{x \rightarrow 0^{-}} f(x) \quad$ (b) $\lim _{x \rightarrow 2^{+}} f(x)$
2. Find (a) $\lim _{x \rightarrow 3} \frac{x^{2}-9}{x^{2}-4 x+3}$
(b) $\lim _{x \rightarrow \infty} x^{2} e^{-x}$


## B. Definition of the Derivative

3. Given $f(x)=\frac{1}{x-1}$ use the definition of the derivative to find $f^{\prime}(x)$.

## C. Graphs

4. Given the graph of $f(x)$, sketch the graph of $f^{\prime}(x)$.
(a)

(b)

5. Sketch the graph of $y=f(x)$ given that $f$ has a horizontal asymptote $y=0$, a vertical asymptote $x=2$, a critical point at $(0,2)$, an inflection point at $(-1,1), f^{\prime}(x)>0$ for $x<0$ and $x>2$, and $f^{\prime \prime}(x)>0$ for $x<-1$.

## D. Rules of Differentiation

6. Differentiate:
(a) $f(x)=x^{2} \arctan (x+3)$
(b) $g(x)=\frac{\sin \left(x^{2}+1\right)}{\ln x}$
(c) $h(x)=\ln x+e^{x}$
(d) $F(x)=(\sin x)^{x}$
(e) given $5 \ln (x+y)=2 x$, find $y^{\prime}$

## E. Linear Approximation / Differentials

7. Find the equation of the tangent line to $y=\tan x$ at $x=\pi / 4$. Use your answer to estimate $\tan (0.8)$.
8. Given $V=\frac{4}{3} \pi r^{3}$ find the differential $d V$. Use your answer to approximate the change in $V$ when $r$ increases from 10 to 10.1.

## F. Distance / Velocity / Acceleration

9. An object is thrown vertically such that its height [in meters] at time $t$ [in seconds] is given by $h(t)=-4.9 t^{2}+49 t$.
(a) Find the velocity and acceleration at $t=3$.
(b) Find the maximum height.
(c) Find the total distance traveled from $t=3$ to $t=6$.

## G. Graph Features

10. Given $f(x)=\frac{x}{x-4}$ find the critical points, inflection points, intervals of increase and decrease, intervals of concavity, and all asymptotes. Sketch a graph of $y=f(x)$.

## H. Max/Min Problems

11. Find the absolute maximum and minimum values of $f(x)=x e^{-x}$ for $-1 \leq x \leq 2$.
12. An open-topped box with a square base has a surface area of 10 square meters. Find the dimensions of the box so that it has the largest possible volume.

## I. Related Rates

13. At noon ship A is 10 km north of ship B. Ship A is traveling north at $30 \mathrm{~km} / \mathrm{h}$, and ship B is traveling east at $20 \mathrm{~km} / \mathrm{h}$. Find the rate at which the distance between them is increasing at 2 PM .

## J. Parametric Curves

14. Given the parametric equations $x=t \sin t$ and $y=t \cos t$ find $\frac{d y}{d x}$ at the point $(\pi / 2,0)$.

## K. Newton's Method

15. Use Newton's Method to estimate the solution of $x-5 \ln x=0$. Using $x_{1}=1.2$, find $x_{2}$ and $x_{3}$.
