MATH 114 – Review Problems

A. Limits

- 1. Refer to the graph to the right. Find: (a) $\lim_{x \to 0^{-}} f(x)$ (b) $\lim_{x \to 2^{+}} f(x)$
- 2. Find (a) $\lim_{x \to 3} \frac{x^2 9}{x^2 4x + 3}$ (b) $\lim_{x \to \infty} x^2 e^{-x}$



B. Definition of the Derivative

3. Given $f(x) = \frac{1}{x-1}$ use the definition of the derivative to find f'(x).

C. Graphs

4. Given the graph of f(x), sketch the graph of f'(x).



5. Sketch the graph of y = f(x) given that f has a horizontal asymptote y = 0, a vertical asymptote x = 2, a critical point at (0, 2), an inflection point at (-1, 1), f'(x) > 0 for x < 0 and x > 2, and f''(x) > 0 for x < -1.

D. Rules of Differentiation

6. Differentiate:

(a) $f(x) = x^2 \arctan(x+3)$ (b) $g(x) = \frac{\sin(x^2+1)}{\ln x}$ (c) $h(x) = \ln x + e^x$ (d) $F(x) = (\sin x)^x$ (e) given $5\ln(x+y) = 2x$, find y'

E. Linear Approximation / Differentials

7. Find the equation of the tangent line to $y = \tan x$ at $x = \pi/4$. Use your answer to estimate $\tan(0.8)$.

8. Given $V = \frac{4}{3}\pi r^3$ find the differential dV. Use your answer to approximate the change in V when r increases from 10 to 10.1.

F. Distance / Velocity / Acceleration

- 9. An object is thrown vertically such that its height [in meters] at time t [in seconds] is given by $h(t) = -4.9t^2 + 49t$.
 - (a) Find the velocity and acceleration at t = 3.
 - (b) Find the maximum height.
 - (c) Find the total distance traveled from t = 3 to t = 6.

G. Graph Features

10. Given $f(x) = \frac{x}{x-4}$ find the critical points, inflection points, intervals of increase and decrease, intervals of concavity, and all asymptotes. Sketch a graph of y = f(x).

H. Max/Min Problems

- 11. Find the absolute maximum and minimum values of $f(x) = xe^{-x}$ for $-1 \le x \le 2$.
- 12. An open-topped box with a square base has a surface area of 10 square meters. Find the dimensions of the box so that it has the largest possible volume.

I. Related Rates

13. At noon ship A is 10 km north of ship B. Ship A is traveling north at 30 km/h, and ship B is traveling east at 20 km/h. Find the rate at which the distance between them is increasing at 2 PM.

J. Parametric Curves

14. Given the parametric equations $x = t \sin t$ and $y = t \cos t$ find $\frac{dy}{dx}$ at the point $(\pi/2, 0)$.

K. Newton's Method

15. Use Newton's Method to estimate the solution of $x - 5 \ln x = 0$. Using $x_1 = 1.2$, find x_2 and x_3 .