MATH 114: Quiz #7 – SOLUTIONS

/5 **Problem 1:** Find the point(s) on the ellipse $4x^2 + y^2 = 4$ that are farthest from the point $(1, 0)$.

We can maximize the distance-squared to the point (x, y):

$$L^2 = (x-1)^2 + y^2.$$

With the constraint that (x, y) must lie on the ellipse we get

$$y^2 = 4 - 4x^2 \implies L^2 = (x-1)^2 + (4 - 4x^2) \equiv f(x).$$

We need to maximize $f(x)$ on the closed interval $[-1, 1]$, so first find critical point(s) of f:

$$f'(x) = 2(x-1) - 8x = -6x - 2$$

$$f'(x) = 0 \implies -6x - 2 = 0 \implies x = -1/3$$

then evaluate and compare:

$$\begin{aligned} &f(-\tfrac{1}{3}) = (-\tfrac{1}{3} - 1)^2 + (4 - 4(\tfrac{1}{3})^2) = \frac{16}{3} = 5\tfrac{1}{3} \\ &f(-1) = 4 \\ &f(1) = 0 \end{aligned}$$

So the points farthest from $(1, 0)$ are

$$\boxed{\left(-\frac{1}{3}, \pm\frac{4\sqrt{2}}{3}\right)}$$

/3 **Problem 2:** Use Newton's method to find $\sqrt[3]{30}$ correct to 4 decimal places.

We can find $\sqrt[3]{30}$ by solving the equation

$$0 = x^3 - 30 \equiv f(x).$$

We have

$$f'(x) = 3x^2$$

so Newton's method gives

$$x_{n+1} = x_n - \frac{x_n^3 - 30}{3x_n^2}.$$

With $x_0 = 3$ (since $3^3 = 27 \approx 30$) we get:

$$\begin{aligned} x_1 &= 3 - \frac{3^3 - 30}{3 \cdot 3^2} \approx 3.11111 \\ x_2 &= x_1 - \frac{x_1^3 - 30}{3x_1^2} \approx 3.10724 \\ x_2 &= x_2 - \frac{x_2^3 - 30}{3x_2^2} \approx 3.10723 \end{aligned}$$

so to 4 decimal places we have

$$\boxed{\sqrt[3]{30} \approx 3.1072}$$