MATH 114: Quiz #6 – SOLUTIONS

/4 **Problem 1:** Gravel is being dumped from a conveyor belt at a rate of $3\,\mathrm{m}^3/\mathrm{min}$ onto a pile whose diameter and height remain equal as the pile grows. How fast is the height of the pile increasing when the pile is $2\,\mathrm{m}$ high?

For a cone with equal diameter and height ($D = 2r = h$) we have

$$V = \frac{\pi}{3} r^2 h = \frac{\pi}{3}\left(\frac{h}{2}\right)^2 h = \frac{\pi}{12} h^3$$

$$\implies V' = \frac{\pi}{12} \cdot 3h^2 \cdot h'$$

$$\implies 3 = \frac{\pi}{12} \cdot 3(2)^2 h'$$

$$\implies \boxed{h' = \frac{3}{\pi} \approx 0.95\,\mathrm{m/min}}$$

/4 **Problem 2:** A certain right-angled triangle has a height known to be exactly $20\,\mathrm{cm}$. The length of the base is measured to be $30\,\mathrm{cm}$ with a possible error of $\pm 0.5\,\mathrm{cm}$. Calculate the length of the hypotenuse, and use differentials to estimate the possible error.

Pythagoras:

$$L^2 = x^2 + y^2 \implies L = \sqrt{x^2 + y^2} = \sqrt{20^2 + 30^2} \approx 36.1\,\mathrm{cm}$$

Error analysis:

$$2L\frac{dL}{dx} = 2x \implies \Delta L \approx \frac{x}{L}\Delta x = \frac{30}{36.1}(0.5) \approx 0.42\,\mathrm{cm}$$

So

$$\boxed{L = 36.1 \pm 0.4\,\mathrm{cm}}$$