MATH 114: Quiz #5 – SOLUTIONS

/4 **Problem 1:** Differentiate:

(a) $f(x) = x \ln(\arctan x)$

$$f'(x) = \ln(\arctan x) + x \cdot \frac{1}{\arctan x} \cdot \frac{1}{1+x^2}$$

(b) $y = (\sqrt{x})^x$

$$\ln y = \ln(\sqrt{x})^x = x \ln \sqrt{x} \implies \frac{y'}{y} = \ln \sqrt{x} + x \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{2} x^{-1/2}$$

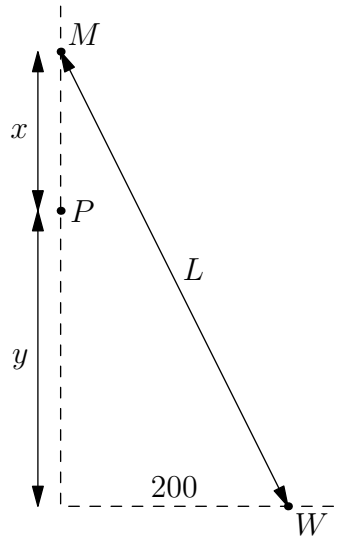
$$\implies y' = (\sqrt{x})^x \left(\ln \sqrt{x} + \frac{1}{2} \right)$$

/4 **Problem 2:** A man starts walking north at 1.2 m/s from a point P. Five minutes later a woman starts walking south at 1.6 m/s from a point 200 m due east of P. At what rate are the people moving apart 15 min after the woman starts walking?

$$L^2 = (x+y)^2 + 200^2$$

$$\implies 2LL' = 2(x+y)(x'+y')$$

$$\implies L' = \frac{(x+y)(x'+y')}{L}$$



At the instant in question we have:

$$x = (20\,\text{min})(60\,\text{s/min})(1.2\,\text{m/s}) = 1440\,\text{m}$$
$$y = (15\,\text{min})(60\,\text{s/min})(1.6\,\text{m/s}) = 1440\,\text{m}$$
$$\implies L = \sqrt{(1440+1440)^2 + 200^2} \approx 2887\,\text{m}$$

So

$$L' \approx \frac{(1440+1440)(1.2+1.6)}{2887} \approx 2.79\,\text{m/s}$$