MATH 114: Quiz #3 – SOLUTIONS

/2 **Problem 1:** Evaluate:

$$\lim_{x\to\infty} \frac{x^2+2}{x^3+x^2-1}$$

$$\lim_{x\to\infty} \frac{x^2+2}{x^3+x^2-1} = \lim_{x\to\infty} \frac{\frac{1}{x}+\frac{2}{x^3}}{1+\frac{1}{x}-\frac{1}{x^3}}$$

$$= \frac{\lim_{x\to\infty}\frac{1}{x}+\lim_{x\to\infty}\frac{2}{x^3}}{\lim_{x\to\infty}1+\lim_{x\to\infty}\frac{1}{x}-\lim_{x\to\infty}\frac{1}{x^3}}$$

$$= \frac{0+0}{1+0+0} = \boxed{0}$$

/2 **Problem 2:** Evaluate:

$$\lim_{x\to-3^-} \frac{x+2}{x+3}$$

$$\lim_{x\to-3^-} \frac{x+2}{x+3} = \frac{-3}{0^-} = \boxed{+\infty}$$

/4 **Problem 3:** State the definition of the derivative $f'(x)$ of a function $f(x)$, and use it to evaluate $f'(x)$ for

$$f(x) = \sqrt{2x}$$

$$\begin{aligned}
f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0}\frac{\sqrt{2(x+h)}-\sqrt{2x}}{h} \\
&= \lim_{h\to 0}\frac{\sqrt{2(x+h)}-\sqrt{2x}}{h}\cdot\frac{\sqrt{2(x+h)}+\sqrt{2x}}{\sqrt{2(x+h)}+\sqrt{2x}} \\
&= \lim_{h\to 0}\frac{2(x+h)-2x}{h\left(\sqrt{2(x+h)}+\sqrt{2x}\right)} \\
&= \lim_{h\to 0}\frac{2}{\sqrt{2(x+h)}+\sqrt{2x}} \\
&= \frac{2}{\sqrt{2(x+0)}+\sqrt{2x}} = \frac{2}{2\sqrt{2x}} = \boxed{\frac{1}{\sqrt{2x}}}
\end{aligned}$$