MATH 114: Quiz #2 – SOLUTIONS

/4 **Problem 1:** Evaluate:

$$\lim_{x\to 16} \frac{4-\sqrt{x}}{16x-x^2}.$$

Justify your solution carefully using limit laws.

$$\begin{aligned}
\lim_{x\to 16} \frac{4-\sqrt{x}}{16x-x^2} &= \lim_{x\to 16} \frac{4-\sqrt{x}}{x(16-x)} \cdot \frac{4+\sqrt{x}}{4+\sqrt{x}} \\
&= \lim_{x\to 16} \frac{16-x}{x(16-x)(4+\sqrt{x})} \\
&= \lim_{x\to 16} \frac{1}{x(4+\sqrt{x})} && \text{(just algebra to this point)} \\
&= \frac{\lim_{x\to 16} 1}{\left(\lim_{x\to 16} x\right) \cdot \lim_{x\to 16} (4+\sqrt{x})} && \text{(by limit laws)} \\
&= \frac{1}{16\left(\lim_{x\to 16} 4 + \lim_{x\to 16} \sqrt{x}\right)} && \text{(limit laws again)} \\
&= \frac{1}{16\left(4+\sqrt{16}\right)} = \boxed{\frac{1}{128}}
\end{aligned}$$

/4 **Problem 2:** For what value(s) of the constant c is the following function continuous?

$$f(x) = \begin{cases} x^2 + cx & \text{if } x < 3 \\ x - c & \text{if } x \geq 3 \end{cases}$$

This function is continuous everywhere except possibly at $x = 3$. For continuity at $x = 3$ we need to have

$$\lim_{x\to 3} f(x) = f(3) = 3 - c$$

so consider:

$$\begin{aligned}
\lim_{x\to 3^-} f(x) &= \lim_{x\to 3^-} (x^2 + cx) = 9 + 3c \\
\lim_{x\to 3^+} f(x) &= \lim_{x\to 3^-} (x - c) = 3 - c.
\end{aligned}$$

Thus $\lim_{x\to 3} f(x)$ exists (and equals $f(3)$) only if

$$9 + 3c = 3 - c \implies 6 = -4c \implies \boxed{c = -\frac{3}{2}}$$