THOMPSON RIVERS UNIVERSITY

MATH 114
Calculus I

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MIDTERM EXAM #2
SOLUTIONS

26 March 2010 12:30–13:20

Instructions:

1. Read all instructions carefully.
2. Read the whole exam before beginning.
3. Make sure you have all 4 pages.
4. Organization and neatness count.
5. You must clearly show your work to receive full credit.
6. You may use the backs of pages for calculations.
7. You may use an approved calculator.

PROBLEM	GRADE	OUT OF
1		8
2		4
3		6
4		6
5		9
TOTAL:		33

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Problem 1: Evaluate the following derivatives:

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(a) $\dfrac{d}{dx}\arcsin(3\sqrt{x})$

$$= \boxed{\frac{1}{\sqrt{1-9x}} \cdot 3 \cdot \tfrac{1}{2}x^{-1/2}}$$

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(b) y' where $y = (\ln x)^{\cos x}$

$$y = \cos x \cdot \ln x \implies \boxed{y' = -\sin x \cdot \ln x + \cos x \cdot \frac{1}{x}}$$

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(c) $\dfrac{dy}{dx}$ where $y^3 + 7y = x^3$.

$$3y^2y' + 7y' = 3x^2 \implies y'(3y^2+7) = 3x^2 \implies \boxed{y' = \frac{3x^2}{3y^2+7}}$$

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Problem 2: The period (in seconds) of a simple pendulum is given by $T = 2\pi\sqrt{L/g}$ where $g = 9.8\,\mathrm{m}^2/\mathrm{s}$ is the acceleration of gravity and L is the length of the pendulum. For a particular pendulum the length is $L = 2.80 \pm 0.05\,\mathrm{m}$. Find the period for this pendulum, and use differentials to estimate the uncertainty.

$$T = 2\pi\left(\frac{L}{g}\right)^{1/2} = 2\pi\left(\frac{2.8}{9.8}\right)^{1/2} \approx 3.36\,\mathrm{s}$$

$$dT = 2\pi \cdot \frac{1}{2}\left(\frac{L}{g}\right)^{-1/2}\frac{1}{g}\,dL = \frac{\pi}{\sqrt{gL}}\,dL$$

$$\implies \Delta T \approx \frac{\pi}{\sqrt{9.8 \cdot 2.8}}(0.05) \approx 0.03\,\mathrm{s}$$

$$\implies \boxed{T = 3.36 \pm 0.03\,\mathrm{s}}$$

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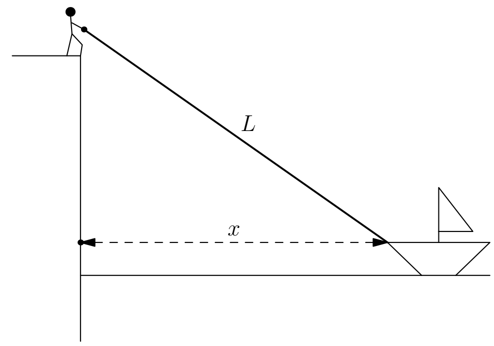
Problem 3: A woman on a dock is pulling in a rope fastened to the bow of a small boat. The woman's hands are 3 m higher than the point where the rope is attached to the boat. If she is retrieving the rope at a rate of 0.5 m/s, how fast is the boat approaching the dock when 5 m of rope is still out?

$$x^2 + 3^2 = L^2 \implies 2xx' = 2LL' \implies x' = \frac{LL'}{x}$$

When $L = 5$: $x^2 + 3^2 = L^2 \implies x = 4$, so

$$x' = \frac{(5)(-0.5)}{4} = \boxed{-0.625\,\mathrm{m/s}}$$



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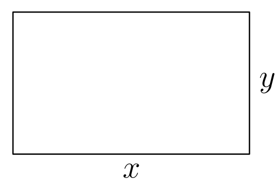
Problem 4: For a rectangle with a perimeter of 12 cm, find the dimensions such that the rectangle has the largest possible area.



- With the constraint
$$2x + 2y = 12 \implies y = 6 - x$$
- we wish to maximize
$$A = xy = x(6-x) = 6x - x^2$$
on the closed interval $[0, 6]$
- Find critical points:
$$A' = 6 - 2x = 0 \implies x = 3$$
- Evaluate and compare (closed interval method):
$$A(0) = 0$$
$$A(6) = 0$$
$$A(3) = 9 \Leftarrow \text{absolute max}$$
- Therefore the optimal dimensions are
$$\boxed{x = y = 3\,\mathrm{cm}}$$

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Problem 5: Consider the function $f(x) = 2x^3 - 3x^2 - 12x + 3$.

(a) Find and classify the critical points of $f(x)$.

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Find critical points:

$$f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x-2)(x+1)$$

$$f'(x) = 0 \implies \boxed{x = -1,\, 2}$$

Use the second derivative test:

$$f''(x) = 12x - 6$$

$$f''(-1) = -18 < 0 \implies \boxed{f \text{ has a local max at } x = -1,\, y = 10}$$

$$f''(2) = 18 > 0 \implies \boxed{f \text{ has a local min at } x = 2,\, y = -17}$$

(b) Make a table of intervals of concavity for $f(x)$ and identify any inflection points.

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Find potential inflection points:

$$f'' = 12x - 6 = 0 \implies x = 1/2$$

interval	f''	f
$(-\infty, 1/2)$	$-$	concave down
$(1/2, \infty)$	$+$	concave up

So $f(x)$ has an inflection point at $\boxed{x = 1/2,\, y = -7/2}$.

(c) Make a careful sketch of the graph of $y = f(x)$.

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