

MATH 114 Calculus I

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MIDTERM EXAM #1 SOLUTIONS

12 February 2010 12:30–13:20

PROBLEM	GRADE	OUT OF
1		8
2		4
3		5
4		13
5		5
6		3
TOTAL:		38

Instructions:

- 1. Read all instructions carefully.
- 2. Read the whole exam before beginning.
- 3. Make sure you have all 5 pages.
- 4. Organization and neatness count.
- 5. You must clearly show your work to receive full credit.
- 6. You may use the backs of pages for calculations.
- 7. You may use an approved calculator.

Problem 1: Evaluate the following limits.

$$\frac{78}{73}$$
 (a) $\lim_{x \to 4} \frac{\sqrt{x-2}}{x-4}$

$$= \lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4} \cdot \frac{\sqrt{x} + 2}{\sqrt{x} + 2}$$
$$= \lim_{x \to 4} \frac{x - 4}{(x - 4)(\sqrt{x} + 2)}$$
$$= \lim_{x \to 4} \frac{1}{\sqrt{x} + 2}$$
$$= \frac{1}{\sqrt{4} + 2} = \boxed{\frac{1}{4}}$$

(b)
$$\lim_{x \to 1^{-}} \frac{x}{\sqrt{1-x^2}}$$

$$=\frac{1}{0^+}=+\infty$$

(c)
$$\lim_{x \to \infty} \frac{3x^3 - 5x^2 + 7}{8 + 2x - 5x^3}$$

$$= \lim_{x \to \infty} \frac{3 - \frac{5}{x} + \frac{7}{x^3}}{\frac{8}{x^2} + \frac{2}{x^2} - 5} = \frac{3 - 0 + 0}{0 + 0 - 5} = \boxed{-\frac{3}{5}}$$

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Problem 2: Consider the function

$$g(x) = \begin{cases} x^2 + 4, & x < 2\\ C, & x = 2\\ x^3, & x > 2. \end{cases}$$

For what value(s) of C is g(x) continuous everywhere. Justify your answer.

We have

$$\lim_{x \to 2^{-}} g(x) = 2^{2} + 4 = 8$$
$$\lim_{x \to 2^{+}} g(x) = 2^{3} = 8$$

 \mathbf{SO}

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 $\lim_{x \to 2} g(x) = 8.$

For g to be continuous at x = 2 we require that

$$\lim_{x \to 2} g(x) = g(2) \implies \boxed{8 = C}$$

Problem 3: (a) State the *definition* of the derivative of a function f(x).

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$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(b) Using your definition in part (a), evaluate the derivative of /3

$$f(x) = x^2 - 3x.$$

$$f'(x) = \lim_{h \to 0} \frac{\left[(x+h)^2 - 3(x+h)\right] - \left[x^2 - 3x\right]}{h}$$
$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h}$$
$$= \lim_{h \to 0} \frac{2xh + h^2 - 3h}{h}$$
$$= \lim_{h \to 0} (2x+h-3) = \boxed{2x-3}$$

Problem 4: Evaluate the following derivatives. /13

$$\frac{13}{3} \quad (a) \quad \frac{d}{dx} \left(x^8 - \sqrt[4]{x} + \frac{2}{x^3} + \pi^4 + x^\pi \right)$$

$$=8x^7 - \frac{1}{4}x^{-3/4} - 6x^{-4} + \pi x^{\pi-1}$$

(b)
$$\frac{d}{dx}\sqrt{x^2+2x-3}$$

$$= \frac{1}{2}(x^2 + 2x - 3)^{-1/2} \cdot (2x + 2)$$

(c)
$$f''(x)$$
 where $f(x) = x \sin 2x$
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$$f'(x) = \sin 2x + 2x \cos 2x$$
$$\implies f''(x) = 2\cos 2x + 2\cos 2x - 4x\sin 2x$$

(d)
$$\frac{d}{dx}\left(\frac{x}{1+\frac{1}{x}}\right)$$

= $\frac{(1)(1+\frac{1}{x})-x(-x^{-2})}{(1+\frac{1}{x})^2}$

Problem 5: Find an equation for the tangent line to the graph of

$$y = x^3 - 5x^2 + 7$$

at the point (2, -5).

The slope of the tangent line is given by:

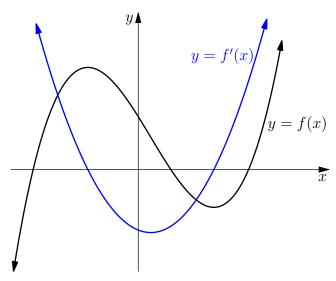
$$y'(x) = 3x^2 - 10x \implies y'(2) = 3(2^2) - 10(20) = -8$$

so the equation of the tangent line is

$$y = y_0 + m(x - x_0)$$

= (-5) + (-8)(x - 2)
 $\implies y = -8x + 11$

Problem 6: The graph of y = f(x) is shown below. On same set of axes, sketch the graph of y = f'(x).



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