THOMPSON RIVERS UNIVERSITY

MATH 114
Calculus I

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MIDTERM EXAM #1
SOLUTIONS

12 February 2010 12:30–13:20

PROBLEM	GRADE	OUT OF
1		8
2		4
3		5
4		13
5		5
6		3
TOTAL:		38

Instructions:

1. Read all instructions carefully.
2. Read the whole exam before beginning.
3. Make sure you have all 5 pages.
4. Organization and neatness count.
5. You must clearly show your work to receive full credit.
6. You may use the backs of pages for calculations.
7. You may use an approved calculator.

/8

Problem 1: Evaluate the following limits.

/3 (a) $\lim_{x\to 4} \dfrac{\sqrt{x}-2}{x-4}$

$$\begin{aligned} &= \lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} \\ &= \lim_{x\to 4} \frac{x-4}{(x-4)(\sqrt{x}+2)} \\ &= \lim_{x\to 4} \frac{1}{\sqrt{x}+2} \\ &= \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}} \end{aligned}$$

/2 (b) $\lim_{x\to 1^-} \dfrac{x}{\sqrt{1-x^2}}$

$$= \frac{1}{0^+} = \boxed{+\infty}$$

/3 (c) $\lim_{x\to\infty} \dfrac{3x^3-5x^2+7}{8+2x-5x^3}$

$$= \lim_{x\to\infty} \frac{3-\frac{5}{x}+\frac{7}{x^3}}{\frac{8}{x^2}+\frac{2}{x^2}-5} = \frac{3-0+0}{0+0-5} = \boxed{-\frac{3}{5}}$$

/4

Problem 2: Consider the function

$$g(x) = \begin{cases} x^2 + 4, & x < 2 \\ C, & x = 2 \\ x^3, & x > 2. \end{cases}$$

For what value(s) of C is $g(x)$ continuous everywhere. Justify your answer.

We have

$$\lim_{x \to 2^-} g(x) = 2^2 + 4 = 8$$
$$\lim_{x \to 2^+} g(x) = 2^3 = 8$$

so

$$\lim_{x \to 2} g(x) = 8.$$

For g to be continuous at $x = 2$ we require that

$$\lim_{x \to 2} g(x) = g(2) \implies \boxed{8 = C}$$

/5

Problem 3: (a) State the *definition* of the derivative of a function $f(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

/3

(b) Using your definition in part (a), evaluate the derivative of

$$f(x) = x^2 - 3x.$$

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{[(x+h)^2 - 3(x+h)] - [x^2 - 3x]}{h} \\ &= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h} \\ &= \lim_{h \to 0} \frac{2xh + h^2 - 3h}{h} \\ &= \lim_{h \to 0} (2x + h - 3) = \boxed{2x - 3} \end{aligned}$$

/13

Problem 4: Evaluate the following derivatives.

/3 (a) $\dfrac{d}{dx}\left(x^8 - \sqrt[4]{x} + \dfrac{2}{x^3} + \pi^4 + x^\pi\right)$

$$= 8x^7 - \tfrac{1}{4}x^{-3/4} - 6x^{-4} + \pi x^{\pi-1}$$

/3 (b) $\dfrac{d}{dx}\sqrt{x^2+2x-3}$

$$= \frac{1}{2}(x^2+2x-3)^{-1/2} \cdot (2x+2)$$

/4 (c) $f''(x)$ where $f(x) = x\sin 2x$

$$f'(x) = \sin 2x + 2x\cos 2x$$

$$\Longrightarrow f''(x) = 2\cos 2x + 2\cos 2x - 4x\sin 2x$$

/3 (d) $\dfrac{d}{dx}\left(\dfrac{x}{1+\frac{1}{x}}\right)$

$$= \frac{(1)(1+\frac{1}{x}) - x(-x^{-2})}{(1+\frac{1}{x})^2}$$

/5

Problem 5: Find an equation for the tangent line to the graph of

$$y = x^3 - 5x^2 + 7$$

at the point $(2, -5)$.

The slope of the tangent line is given by:

$$y'(x) = 3x^2 - 10x \implies y'(2) = 3(2^2) - 10(20) = -8$$

so the equation of the tangent line is

$$\begin{aligned} y &= y_0 + m(x - x_0) \\ &= (-5) + (-8)(x - 2) \end{aligned}$$

$$\implies \boxed{y = -8x + 11}$$

/3

Problem 6: The graph of $y = f(x)$ is shown below. On same set of axes, sketch the graph of $y = f'(x)$.