Thompson Rivers University

MATH 114
Calculus I

Instructor: Richard Taylor

MIDTERM EXAM #1
SOLUTIONS

13 Feb. 2007 11:30–13:20

Instructions:

1. Read all instructions carefully.
2. Read the whole exam before beginning.
3. Make sure you have all 6 pages.
4. Organization and neatness count.
5. You must clearly show your work to receive full credit.
6. You may use the backs of pages for calculations.
7. You may use an approved calculator.

PROBLEM	GRADE	OUT OF
1		12
2		5
3		4
4		6
5		6
6		4
7		12
TOTAL:		49

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Problem 1: Evaluate the following limits. Use the symbols $+\infty$ and $-\infty$ where appropriate.

(a) $\displaystyle\lim_{x\to 2} \frac{x^2+2x-8}{x-2}$

SOLUTION:

$$\lim_{x\to 2} \frac{x^2+2x-8}{x-2} = \lim_{x\to 2} \frac{(x+4)(x-2)}{x-2} = \lim_{x\to 2}(x+4) = \boxed{6}$$

(b) $\displaystyle\lim_{x\to 4} \frac{\frac{2}{x}-\frac{1}{2}}{4-x}$

SOLUTION:

$$\lim_{x\to 4} \frac{\frac{2}{x}-\frac{1}{2}}{4-x} = \lim_{x\to 4} \frac{\frac{4}{2x}-\frac{x}{2x}}{4-x} = \lim_{x\to 4} \frac{4-x}{2x}\frac{1}{4-x} = \lim_{x\to 4}\frac{1}{2x} = \frac{\lim_{x\to 4} 1}{\lim_{x\to 4} 2x} = \boxed{\frac{1}{8}}$$

(c) $\displaystyle\lim_{x\to \infty} \frac{4x^4+x^2}{2x^4+x-8}$

SOLUTION:

$$\lim_{x\to\infty} \frac{4x^4+x^2}{2x^4+x-8} = \lim_{x\to\infty} \frac{4+\frac{1}{x^2}}{2+\frac{1}{x^3}-\frac{8}{x^4}} = \frac{\lim_{x\to\infty} 4+\dfrac{1}{x^2}}{\lim_{x\to\infty} 2+\dfrac{1}{x^3}-\dfrac{8}{x^4}} = \frac{4+0}{2+0-0} = \boxed{2}$$

(d) $\displaystyle\lim_{x\to 2^-} \frac{(x+3)(x-4)}{(x-2)(x-5)}$

SOLUTION:

$$\lim_{x\to 2^-} \frac{(x+3)(x-4)}{(x-2)(x-5)} = \frac{(5)(-2)}{(0^-)(-3)} = \boxed{-\infty}$$

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Problem 2: Consider the following piecewise function.

$$f(x) = \begin{cases} x + a & \text{if } x < 1 \\ x^2 - 2 & \text{if } x \geq 1 \end{cases}$$

(a) For what value(s) of a is $f(x)$ continuous?

SOLUTION:

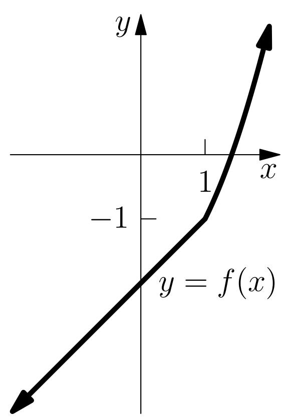
$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x + a) = 1 + a$$
$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x^2 - 2) = -1$$

So $\lim\limits_{x \to 1} f(x) = f(1)$ if $1 + a = -1$

$$\implies \boxed{a = -2}$$

(b) Sketch the graph of $y = f(x)$ for the value of a you found in part (a).

SOLUTION:



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Problem 3: For each of the following, at what numbers is the function f discontinuous?

(a) $f(x) = \dfrac{x+2}{x^2 - x - 6}$

SOLUTION:

$f(x) = \dfrac{x+2}{(x+2)(x-3)}$ is discontinuous at $\boxed{x = -2 \text{ and } x = 3}$.

(b) $f(x) = x \ln |x - 4|$

SOLUTION:

$f(x)$ is discontinuous where $x - 4 = 0 \implies \boxed{x = 4}$.

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Problem 4: (a) State the definition of the derivative, $f'(x)$.

SOLUTION:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

(b) Using your definition from part (a), find the derivative of $f(x) = 5\sqrt{x}$.

SOLUTION:

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{5\sqrt{x+h} - 5\sqrt{x}}{h} \\ &= 5\lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \\ &= 5\lim_{h\to 0} \frac{(x+h) - (x)}{h} \cdot \frac{1}{\sqrt{x+h} + \sqrt{x}} \\ &= 5\lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} \\ &= 5\frac{\lim_{h\to 0} 1}{\lim_{h\to 0} \sqrt{x+h} + \lim_{h\to 0} \sqrt{x}} \\ &= 5\frac{1}{\sqrt{x} + \sqrt{x}} \\ &= \boxed{\frac{5}{2\sqrt{x}}} \end{aligned}$$

(c) What is the slope of the tangent line to the graph of $y = 5\sqrt{x}$ at $x = 25$?

SOLUTION:

$$\text{slope} = f'(25) = \frac{5}{2\sqrt{25}} = \boxed{\frac{1}{2}}$$

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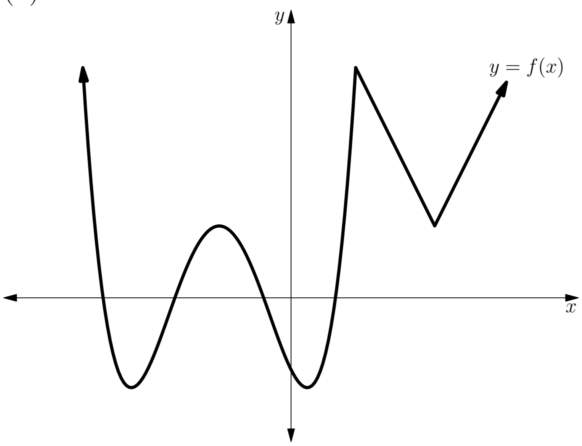
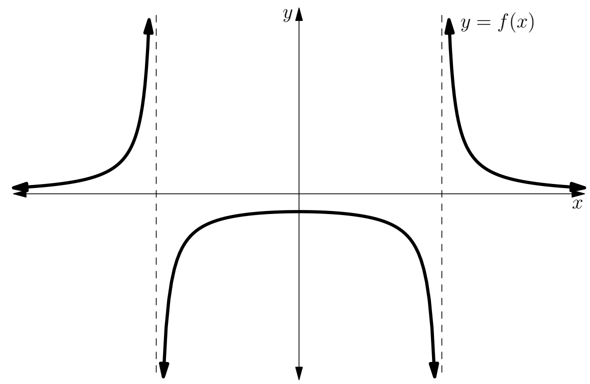
Problem 5: For each the following graphs of $f(x)$, sketch the graph of $f'(x)$ immediately below.

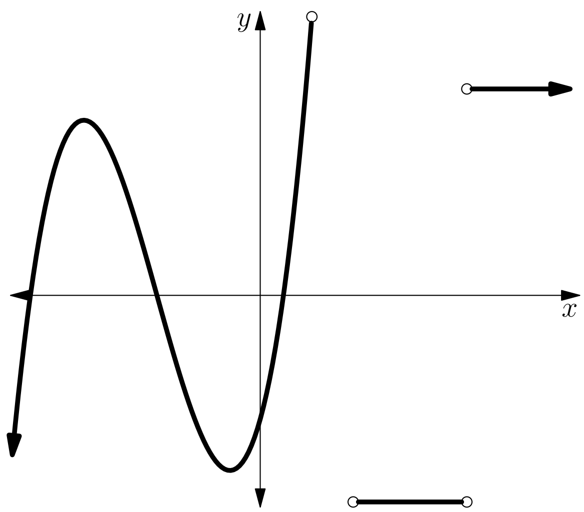
(a)

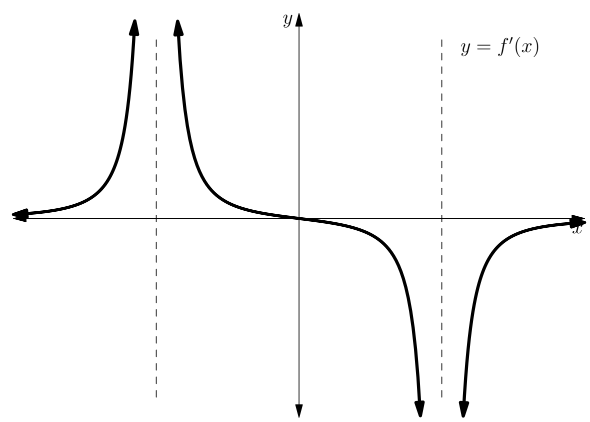


(b)



SOLUTION:





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Problem 6: A particle moves along a straight line. Its position (i.e. its displacement from the origin) at time t is given by the function $f(t) = 3t^2 - 6t$.

(a) What is the particle's instantaneous velocity at time $t = 3$?

SOLUTION:

velocity $v(t) = f'(t) = 6t - 6 \ldots v(3) = 6 \cdot 3 - 6 = \boxed{12}$

(b) At what time(s) is the particle not moving?

SOLUTION:

$v(t) = 0 \implies 6t - 6 = 0 \implies \boxed{t = 1}$

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Problem 7: Find the derivative of each of the following functions. *Do not simplify.*

(a) $f(x) = 4x^2 - \dfrac{1}{x} + x^\pi - \pi^5$

SOLUTION:

$$f(x) = 4x^2 - x^{-1} + x^\pi - \pi^5$$

$$\implies f'(x) = 8x + x^{-2} + \pi x^{\pi - 1}$$

(b) $g(x) = \dfrac{x+2}{x-1}$

SOLUTION:

$$\text{quotient rule:} \quad g'(x) = \boxed{\frac{(1)(x-1) - (1)(x+2)}{(x-1)^2}}$$

(c) $h(x) = x^2 e^x$

SOLUTION:

$$\text{product rule:} \quad h'(x) = \boxed{2xe^x + x^2 e^x}$$

(d) $F(x) = \sqrt[4]{1 + 2x + x^3}$

SOLUTION:

$$F(x) = (1 + 2x + x^3)^{1/4}$$

$$\text{chain rule:} \quad F'(x) = \tfrac{1}{4}(1 + 2x + x^3)^{-3/4}(2 + 3x^2)$$