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Problem 1: Evaluate the following limits.

(a) $\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x+2)(x-2)}{x-2}$

$= \lim_{x\to 2} (x+2)$

$= \boxed{4}$

(b) $\lim_{x\to\infty} \frac{3x^2+x+1}{2x^2-1} = \lim_{x\to\infty} \frac{3+\frac{1}{x}+\frac{1}{x^2}}{2-\frac{1}{x^2}}$

$= \frac{\lim_{x\to\infty}\left(3+\frac{1}{x}+\frac{1}{x^2}\right)}{\lim_{x\to\infty}\left(2-\frac{1}{x^2}\right)}$

$= \frac{3+0+0}{2-0}$

$= \boxed{3/2}$

(c) $\lim_{x\to 0} \frac{\sqrt{1-x}-\sqrt{1+x}}{x} \cdot \frac{\sqrt{1-x}+\sqrt{1+x}}{\sqrt{1-x}+\sqrt{1+x}}$

$= \lim_{x\to 0} \frac{(1-x)-(1+x)}{x(\sqrt{1-x}+\sqrt{1+x})}$

$= \lim_{x\to 0} \frac{-2x}{x(\sqrt{1-x}+\sqrt{1+x})}$

$= \lim_{x\to 0} \frac{-2}{\sqrt{1-x}+\sqrt{1+x}}$

$= \frac{-2}{\sqrt{1-0}+\sqrt{1+0}} = \boxed{-2}$

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Problem 2: Consider the following piecewise function.

$$f(x) = \begin{cases} x^2 + 2 & \text{if } x < 0 \\ \sqrt{x+a} & \text{if } x \geq 0 \end{cases}$$

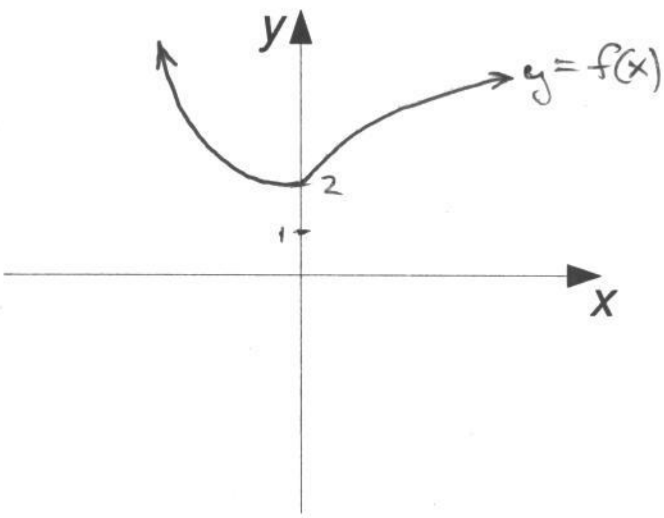
(a) For what value(s) of a is $f(x)$ continuous?

- need $\lim_{x \to 0} f(x) = f(0) = \sqrt{a}$
- $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x^2 + 2 = 2$
- $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \sqrt{x+a} = \sqrt{a}$

must agree, so $\lim_{x \to 0} f(x)$ exists

- need $2 = \sqrt{a}$, $\therefore$ $\boxed{a = 4}$

(b) Sketch the graph of $f(x)$ for the value of a you found in part (a).



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Problem 3: At what value(s) of x are the following functions discontinuous?

(a) $f(x) = x^2 \sin(1/x)$

$\boxed{x = 0}$

(b) $f(x) = \sqrt{\dfrac{1}{|x^2 - 1|}}$ $\quad x^2 - 1 = 0 \rightarrow \boxed{x = \pm 1}$

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Problem 4: (a) State the definition of the derivative, $f'(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

(b) Using your definition from part (a), calculate the derivative of $f(x) = \dfrac{1}{x^2}$.

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{x^2 - (x+h)^2}{(x+h)^2 x^2} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{x^2 - x^2 - 2xh - h^2}{(x+h)^2 x^2 h}$$

$$= \lim_{h\to 0} \frac{-2xh - h^2}{(x+h)^2 x^2 h}$$

$$= \lim_{h\to 0} \frac{-2x - h}{(x+h)^2 x^2}$$

$$= \frac{\lim_{h\to 0} (-2x - h)}{\lim_{h\to 0} (x+h)^2 x^2} = \frac{-2x}{x^2 x^2} = \boxed{\frac{-2}{x^3}}$$

(c) What is the slope of the tangent line to the graph of $y = \dfrac{1}{x^2}$ at $x = 2$?

$$\text{slope} = f'(2) = \frac{-2}{2^3} = \boxed{\frac{-1}{4}}$$

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Problem 5: Calculate the derivative of the following functions. *Do not simplify.*

(a) $f(x) = (x+1)^2(3x+1)^3$

$$f'(x) = 2(x+1)(3x+1)^3 + (x+1)^2 \cdot 3(3x+1)^2 \cdot 3$$

$$= 2(x+1)(3x+1)^3 + 9(x+1)^2(3x+1)^2$$

(b) $f(x) = \sqrt{e^x + \sqrt{x}} = \left(e^x + x^{1/2}\right)^{1/2}$

$$f'(x) = \frac{1}{2}\left(e^x + x^{1/2}\right)^{-1/2} \cdot \left(e^x + \frac{1}{2}x^{-1/2}\right)$$

$$= \frac{e^x + \frac{1}{2\sqrt{x}}}{2\sqrt{e^x + \sqrt{x}}}$$

(c) $f(x) = \sin\left(\dfrac{x+1}{x-1}\right)$

$$f'(x) = \cos\left(\frac{x+1}{x-1}\right) \cdot \frac{(1)(x-1) - (1)(x+1)}{(x-1)^2}$$

$$= \cos\left(\frac{x+1}{x-1}\right) \cdot \frac{-2}{(x-1)^2}$$

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Problem 6: Given the graph of $f(x)$ below, sketch the graph of $f'(x)$.