Name: ______________________________ Student #: ______________

THOMPSON RIVERS UNIVERSITY

MATH 114
Calculus I

S01 – Richard Taylor

FINAL EXAM

26 April 2007 14:00–17:00 Gym M

PROBLEM	GRADE	OUT OF
1		3
2		5
3		10
4		4
5		4
6		6
7		4
8		12
9		4
10		5
11		5
12		5
13		4
14		6
15		5
16		7
17		3
18		8
TOTAL:		100

Instructions:

1. Read all instructions carefully.
2. Read the whole exam before beginning.
3. Make sure you have all 11 pages.
4. Organization and neatness count.
5. You must clearly show your work to receive full credit.
6. You may use the backs of pages for calculations.
7. You may use an approved calculator.

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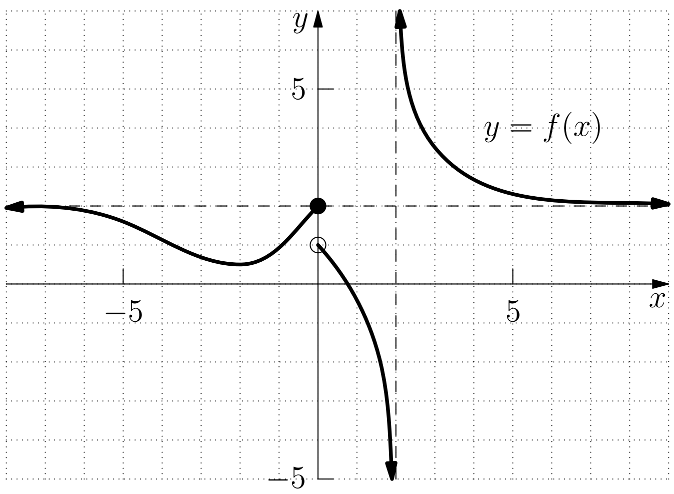
Problem 1: The graph of the function f is shown below. Use the graph to evaluate the following:



(a) $f(0) =$

(b) $\lim_{x\to 0^+} f(x) =$

(c) $\lim_{x\to 2^-} f(x) =$

(d) $\lim_{x\to 3} f(x) =$

(e) $\lim_{x\to -\infty} f(x) =$

(f) $\lim_{x\to +\infty} f'(x) =$

/5

Problem 2: (a) State the *definition* (i.e. using limits) of the derivative of a function f.

(b) Use the *definition* of the derivative to find $f'(x)$ for the function $f(x) = 3x^2 - 8x$.

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Problem 3: Evaluate the following limits.

(a) $\displaystyle\lim_{x \to -5} \frac{x^2 + 2x - 15}{2x^2 + 10x}$

(b) $\displaystyle\lim_{x \to 0} \frac{3 - 3\cos x}{x}$

(c) $\displaystyle\lim_{x \to +\infty} \frac{8 - 3x^2}{x^2 - 2x - 3}$

(d) $\displaystyle\lim_{x \to 2^+} \frac{2x - 5}{(x - 2)^3}$

(e) $\displaystyle\lim_{x \to +\infty} xe^{-x}$

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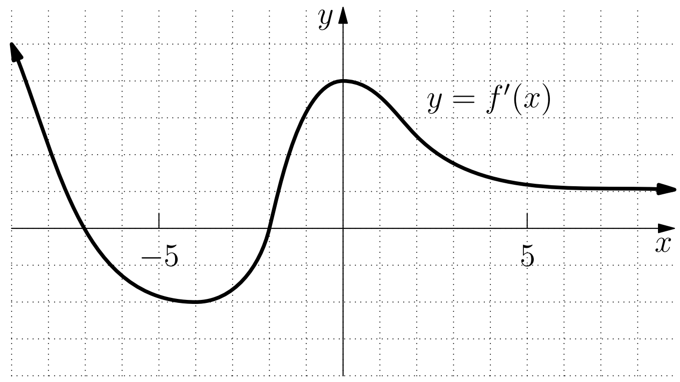
Problem 4: For a certain function f, the graph of its derivative, $f'(x)$, is shown below.



Answer the following questions about f (*not* about f').

(a) On what interval(s) is f decreasing?

(b) On what interval(s) is f concave up?

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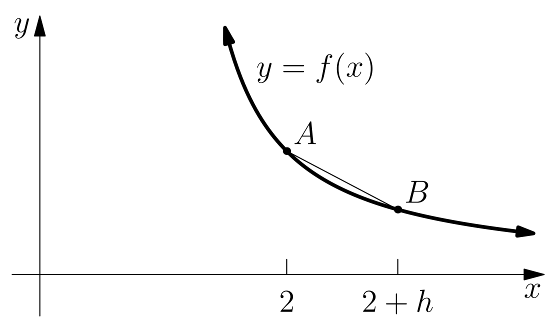
Problem 5: A portion of the graph of the function $f(x) = \dfrac{1}{x-1}$ is shown below.



(a) Find and *simplify* an expression for the slope of the line segment $\overline{AB}$.

(b) Use your answer to (a) to find the slope of the line tangent to the graph at the point A.

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Problem 6: For a certain function f the following is known:

the domain is $(-\infty, \infty)$ $\quad f'(x) = \dfrac{1-x^2}{(1+x^2)^2}$ $\quad f''(x) = \dfrac{2x(x^2-3)}{(1+x^2)^3}$

(a) Find the intervals of increase and decrease for f.

(b) Find the intervals of concavity for f.

/4

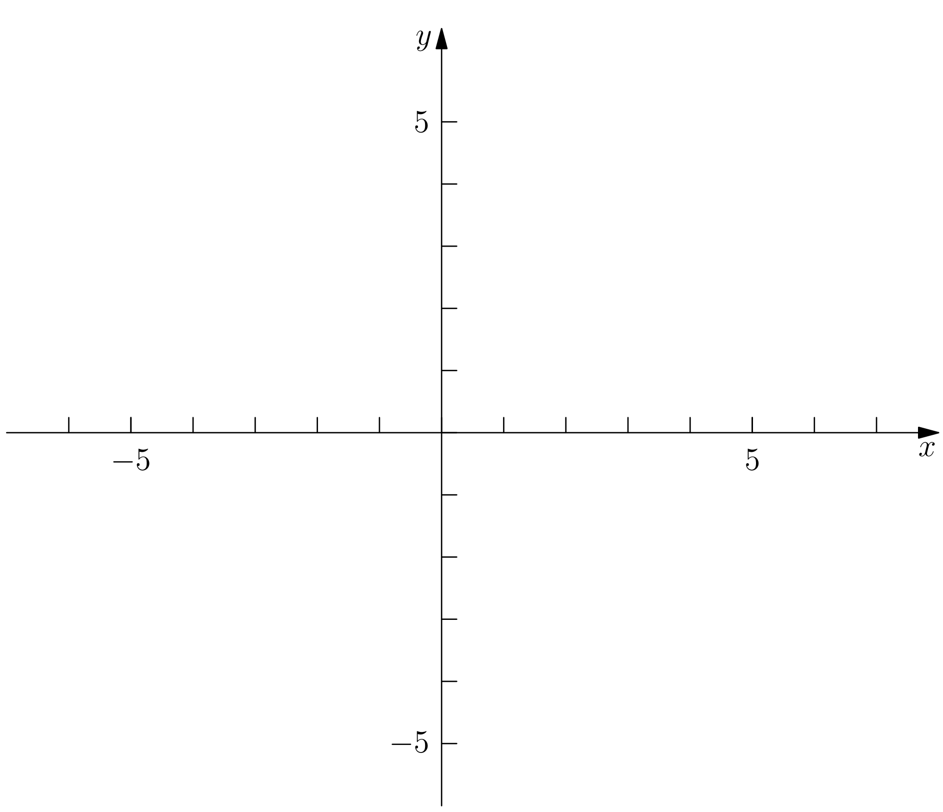
Problem 7: Sketch a possible graph of a continuous function f that has the following properties:

$f(-1) = 2,\ f(2) = 4,\ f(6) = 0$

$f'(x) > 0$ if $x < 2$ or $x > 5$ $\qquad f'(x) < 0$ if $2 < x < 5$

$f''(x) > 0$ if $x < -1$ $\qquad f''(x) < 0$ if $-1 < x < 5$ or $x > 5$

$\lim\limits_{x \to -\infty} f(x) = -2$



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Problem 8: Find the indicated derivatives:

(a) y' if $y = \dfrac{2}{x} + \sqrt[5]{x} - e^x + \pi^2$

(b) $\dfrac{dz}{dt}$ if $z = \dfrac{e^{t^2}}{t^2 - 3}$

(c) $f'(x)$ if $f(x) = (x + \arcsin x)^5$

(d) $g'(x)$ if $g(x) = (\ln x)(\tan(2x))$

/4

Problem 9: The total sales (in thousands of dollars) of a product after t months on the market is given by

$$S(t) = \sqrt{t^2 + 7}; \quad t \geq 0.$$

(a) Calculate $S(3)$ and $S'(3)$.

(b) In words, interpret your answers to part (a). Include appropriate units in your answer.

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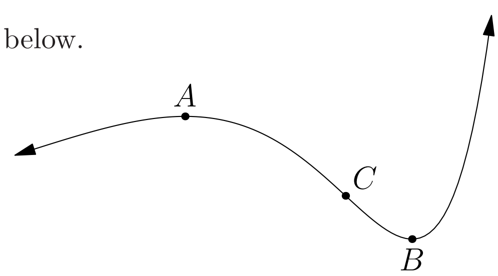
Problem 10: A portion of the graph of $f(x) = x^2 e^x$ is shown below.

(a) Find the x-coordinates of the critical points at A and B.



(b) Find the x-coordinate of the inflection point at C.

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Problem 11: Use implicit differentiation to find an equation of the line tangent to the curve

$$5x^2y + 11\cos y = e^{x-2} + 5x$$

at the point $(2, 0)$.

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Problem 12: For the function $f(x) = \sqrt[3]{x}$

(a) find the equation of the tangent line to the graph of f at the point $(8, 2)$.

(b) use your answer to part (a) to estimate $\sqrt[3]{8.06}$.

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Problem 13: Use logarithmic differentiation to find $\dfrac{dy}{dx}$ if $y = (\arctan x)^x$.

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Problem 14: An antibiotic is introduced to a bacterial culture. The number of bacteria remaining after t days is given by the function

$$N(t) = 20t^3 - 210t^2 + 360t + 1100; \quad 0 \le t \le 7.$$

(a) Find the absolute maximum and minimum number of bacteria over the 7-day period.

(b) Find the time at which the number of bacteria is decreasing at the *fastest rate*.

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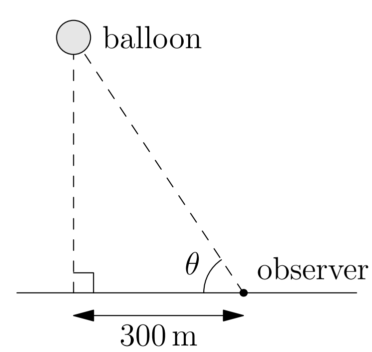
Problem 15: A weather balloon leaves the ground 300 m from an observer and rises vertically at a speed of 12 m/s. How fast is the angle, θ, of the line of sight changing when the balloon is 400 m high?



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Problem 16: The displacement (in cm) at time t (in seconds) of a particle moving horizontally along a straight line is given by

$$x(t) = -3t^3 + 22.5t^2 - 36t + 20; \quad t \geq 0$$

(a) Find the velocity and the acceleration at $t = 3$.

(b) Describe the motion of the particle at $t = 3$. (Is it moving to the right or to the left? Is it speeding up or slowing down? Explain.)

(c) Find the total distance traveled by the particle between $t = 0$ and $t = 3$.

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Problem 17: The parametric equations for a given curve are

$$x(t) = 2 + \cos(t + \tfrac{\pi}{2})$$
$$y(t) = t + e^t$$

Find the slope of the line tangent to the curve at the point where $t = 0$.

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Problem 18: A closed cylindrical can must have a volume of $1000\,\mathrm{cm}^3$. The materials to make the can cost $\$5/\mathrm{cm}^2$ for the top and bottom, and $\$2/\mathrm{cm}^2$ for the remaining (curved) part of the can. What is the minimum possible cost for the materials to make such a can?

The volume of a cylinder is $V = \pi r^2 h$.

The surface area of a closed cylinder is $A = 2\pi rh + 2\pi r^2$.