Name: ____________________ Student #: ________________

THOMPSON RIVERS UNIVERSITY

MATH 114
Calculus I

Instructor: Richard Taylor

FINAL EXAM

6 December 2006 09:00–12:00

PROBLEM	GRADE	OUT OF
1		3
2		4
3		6
4		3
5		3
6		4
7		10
8		4
9		4
10		6
11		4
12		4
13		7
14		4
15		5
16		4
17		5
18		6
TOTAL:		86

Instructions:

1. Read all instructions carefully.
2. Read the whole exam before beginning.
3. Make sure you have all 12 pages.
4. Organization and neatness count.
5. You must clearly show your work to receive full credit.
6. You may use the backs of pages for calculations.
7. You may use an approved calculator.

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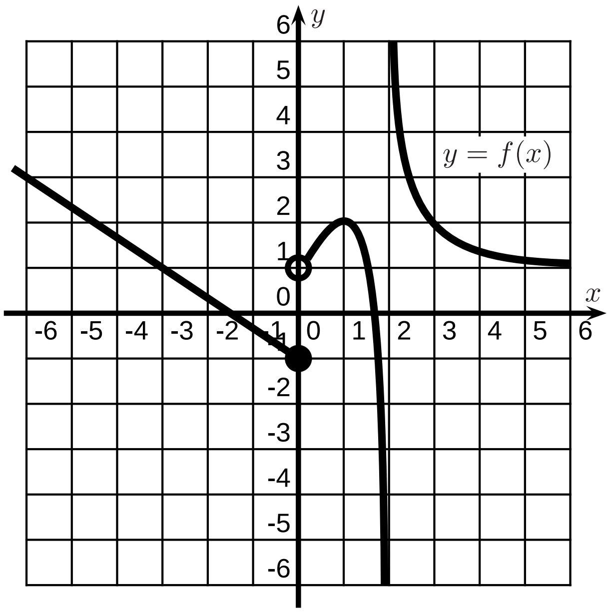
Problem 1: Use the graph of $y = f(x)$ below to answer the following questions.



(a) $f(0) =$

(b) $f'(-4) =$

(c) $\lim_{x\to 0^+} f(x) =$

(d) $\lim_{x\to 2^-} f(x) =$

(e) $\lim_{x\to \infty} f(x) =$

(f) $\lim_{x\to \infty} f'(x) =$

/4

Problem 2: The total sales (in thousands of dollars) of a product after t months on the market is given by $S(t) = \sqrt{t^2 + 7}$ for $t \geq 0$.

(a) Calculate $S(3)$ and $S'(3)$.

(b) Interpret your results from (a). Include all appropriate units.

/6 **Problem 3:** Evaluate the following limits. Use the symbols ∞ and $-\infty$ where appropriate. Do not use L'Hôpital's Rule.

(a) $\displaystyle\lim_{x\to 3} \frac{x^2-9}{x^2-13x+30}$

(b) $\displaystyle\lim_{x\to -\infty} \frac{2x^2-3x+7}{5-4x^2}$

(c) $\displaystyle\lim_{x\to 2^-} \frac{(x-5)(x+3)}{x-2}$

/3 **Problem 4:** Use L'Hôpital's Rule to evaluate $\displaystyle\lim_{\theta\to\pi} \frac{1+\cos\theta}{\sin^2\theta}$.

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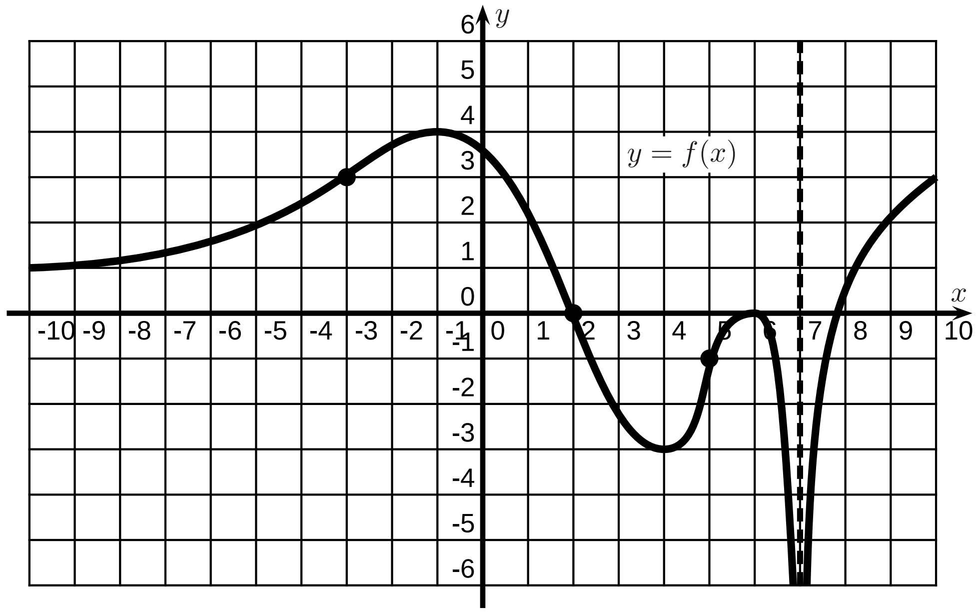
Problem 5: Given the following graph of f, sketch the graph of f' on the same set of axes.



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Problem 6: For the function $f(x) = \dfrac{2}{3x+2}$, find the value of $f'(2)$ using the **definition** of the derivative (i.e. using **limits**).

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Problem 7: Find the following derivatives. **Do not** simplify your answers.

(a) $\dfrac{dy}{dx}$ if $y = \dfrac{x}{3x^2 - 1}$

(b) $f'(x)$ if $f(x) = \arcsin(3\sqrt{x})$

(c) $\dfrac{d}{dt}\left[\sin\left(e^{t^2}\right)\right]$

(d) $\dfrac{d}{d\theta}\left[(2\theta + 1)^2 \tan\theta\right]$

(e) $\dfrac{d^2y}{dx^2}$ if $y = x^4 + 4\ln x + \pi^5$

/4 **Problem 8:** Use implicit differentiation to find an equation for the tangent line to the curve $5x^2y + y^3 = x^4 + 17$ at the point $(1, 2)$.

/4 **Problem 9:** Suppose that a function f has a derivative $f'(x) = \sqrt{x^3 + 1}$ and that $f(2) = 5$.

(a) Use a linear approximation for f near the point where $x = 2$ to estimate the value of $f(1.9)$.

(b) By considering the second derivative of f, determine whether your estimate in part (a) is higher or lower than the actual value of $f(1.9)$.

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Problem 10: The displacement (in cm) at time t (in seconds) of a particle moving along a straight line is given by

$$x(t) = -t^3 + 6t^2 - 9t - 5 \quad \text{for } t \geq 0.$$

(a) What is the particle's instantaneous velocity at $t = 2$?

(b) During what time interval(s) is the particle moving to the right?

(c) Find the time when the velocity is a maximum. Verify that the velocity at this time is indeed a maximum and not a minumum.

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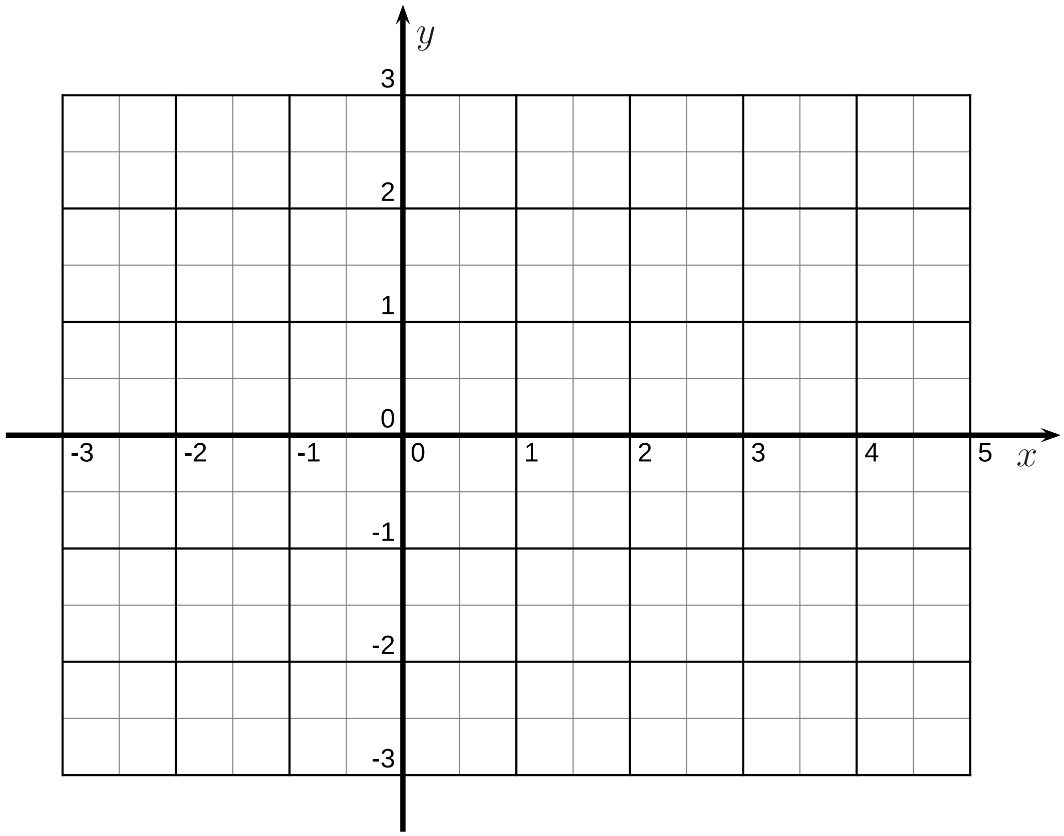
Problem 11: Sketch the graph of a function f with the following properties:

$f(0) = -1$	$\lim_{x\to\pm\infty} f(x) = 1$	$f'(x) > 0$ for $0 < x < 2$ and $x > 2$
$f(-1) = 0$	$\lim_{x\to 2^+} f(x) = -\infty$	$f'(x) < 0$ for $x < 0$
	$\lim_{x\to 2^-} f(x) = +\infty$	$f''(x) > 0$ for $-1 < x < 2$
		$f''(x) < 0$ for $x < -1$ and $x > 2$



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Problem 12: Use logarithmic differentiation to find $\dfrac{dy}{dx}$ if $y = (\arctan x)^{\sqrt{x}}$.

/7 **Problem 13:** Let $f(x) = x^2 - \frac{16}{x}$. Use calculus to answer the following questions:

(a) What is the domain of f?

(b) Find all critical points for f.

(c) Find the intervals of increase and decrease for f.

(d) Final all local maximum and minimum values for f. (For each point where a local maximum or minimum occurs, give values for both x and $f(x)$ and state whether the point is a maximum or minimum).

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Problem 14: For the function $f(x) = e^{-x^2}$ answer the following questions. USE EXACT VALUES.

(a) Find the intervals of concavity for f.

(b) Give the (x, y) coordinates of any inflection points on the graph of f.

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Problem 15: Find the absolute maximum and minimum values of $f(x) = x + 2\cos x$ on the interval $[0, \pi]$. Use exact values for critical points where possible.

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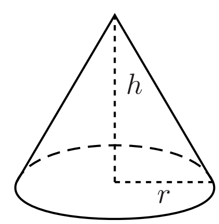
Problem 16: Find an equation for the line tangent to the parametric curve

$$x(t) = 2 + \cos(t + \pi/2), \quad y(t) = t + e^t$$

at the point where $t = 0$.

/5

Problem 17: Sand pours from a conveyor belt at a rate of $3\,\text{m}^3/\text{min}$, forming a conical pile. The height of the pile is always 2 times the radius of the circular base. How fast is the height changing when the volume is $25\,\text{m}^3$? (Recall that the volume of a cone is $V = \frac{1}{3}\pi r^2 h$ where h is the height and r is the radius of the base.)



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Problem 18: A freshwater pipeline is to be run from a source on land to a resort community on an island in the ocean 8 km offshore and 20 km along the shore, as shown in the diagram below. The cost of laying pipe is \$60 per meter over land and \$90 per meter under the ocean. If the total cost of the pipeline is to be minimized, how far should the pipeline run along shore? What is the minimum cost?