Name: $\qquad$
Student \#: $\qquad$

# Midterm Exam \#2 

MATH 112 - Calculus I<br>Okanagan University College

18 November 2003

## Instructions:

1. Read all instructions carefully.
2. Read the whole exam before beginning; make sure you have all 6 pages.
3. Write your name on each page (in case the exam pages become separated).
4. Final answers are not sufficient for full credit-you must clearly present and justify your solution method. Part marks will be awarded if you don't obtain the final answer.
5. If you run out of room, continue your solution on the back of the page.

Problem 1(a): Find the derivative of $f(x)=\arctan \left(x^{2}\right)$.

Problem 1(b): Find the derivative of $g(w)=\sqrt{\arcsin (w)}$
$\qquad$

Problem 2(a): The set of points satisfying the equation

$$
x^{4}+x y+y^{3}=3
$$

implicitly defines a function $y=f(x)$. The equation's graph is shown below. The point $(1,1)$ is on the graph (you can easily check this). Find an equation for the line tangent to the graph at this point.

Problem 2(b): For $f(x)$ as defined in part (a), use a linear approximation to estimate $f(0.9)$. That is, approximate the point on the graph at $x=0.9$.

Problem 2(c): The graph shows that the equation $f(x)=0$ has a solution near $x=1.3$. Suppose Newton's method is used to find this solution, using an initial approximation $x_{1}=1$. Indicate, on the graph above, where the second approximation $x_{2}$ would be located on the $x$-axis. You don't need to do any calculations.

Problem 3: Use Newton's method to find a solution of

$$
\cos x=x^{3}
$$

accurate to 2 decimal places. (Make sure your calculator is in radians!)

Problem 4: Use a linear approximation to estimate the value of $\sqrt{15}$.
$\qquad$

Problem 5: A common method for estimating the height of a tall object (e.g., a tree) is illustrated in the diagram below. A protractor is used to measure the angle of elevation, $\theta$, to the top of the tree from a point on the ground a certain distance away. Suppose one stands 10 m from the base of a tree and finds the angle from the ground to the top of the tree is $\theta=1.2$ radians, accurate to within 0.1 radians. What is the height of the tree, $h$, and how big is the uncertainty in this value?

Problem 6: An $10,000 \mathrm{~m}^{3}$ oil spill spreads out in a circle on the surface of the water. An observer in a helicopter finds, at a particular instant, that the diameter of the spill is 2.5 km and growing at $0.1 \mathrm{~km} / \mathrm{h}$. Assuming the oil patch has uniform thickness, $h$, what is this thickness and how quickly is it decreasing?

Problem 7: 100 m of fencing is to be used to enclose two separate areas, one circular and one square. What should the dimensions of the two areas be so as to maximize the total enclosed area?

