Name: ________________________

GRADE: /50

Student #: ____________________

Midterm Exam #1

MATH 112 – Calculus I

Okanagan University College
21 October 2003

Instructions:

1. *Read all instructions carefully.*
2. *Read the whole exam before beginning*; make sure you have all 6 pages.
3. Write your name on each page (in case the exam pages become separated).
4. Organize and write your solutions **neatly**. If you run out of room, continue your solution on the back of the page.
5. Show your work, and explain yourself: part marks will be awarded even if you don't obtain the final answer.

/4

Problem 1: Evaluate the following limit:

$$\lim_{x\to 2} \frac{x^2 + 5x - 14}{x^2 - 4}.$$

/2

Problem 2(a): State the definition of the derivative, $f'(x)$.

/4

Problem 2(b): Use the *definition* of $f'(x)$ to compute the derivative of

$$f(x) = \frac{2}{x}.$$

/4

Problem 3(a): Compute the derivative of $g(x) = (x+3)^2 \cos(x)$.

/4

Problem 3(b): For $g(x)$ as given in problem 3(a), find an equation for the tangent line to the graph of $y = g(x)$ at $x = 0$.

/4

Problem 3(c): Compute the derivative of $h(t) = \dfrac{\sqrt{t^2+1}}{4t^3 - t}$.

/4

Problem 3(d): Compute the derivative of $F(w) = e^{\cos(w^2)}$.

/3

Problem 4(a): Suppose $f(x)$ is defined by

$$f(x) = \begin{cases} mx + b & \text{if } x \leq 0 \\ 3x^2 + 1 & \text{if } x > 0 \end{cases}$$

where m and b are constants. For what value of b is $f(x)$ a continuous function? (Justify your answer.)

/3

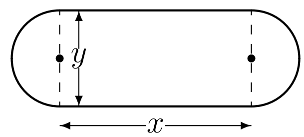
Problem 4(b): Let b have the value you found in problem 6(a). For what value of m is $f(x)$ a *differentiable* function?

/3

Problem 4(c): Sketch a graph of $y = f(x)$, for the values of m and b you found in parts (a) and (b).

/8

Problem 5: An architect wishes to design a room as shown in the diagram (a rectangular region with two opposing walls replaced by semicircles). The area of the room must be 3π (meters squared). Flat walls cost \$100 per linear meter to build, and curved walls cost \$200 per linear meter. How should the dimensions of the room be chosen to minimize the cost of building the walls?



/4

Problem 6(a): A bicycle is travelling in a straight line. Its distance from its starting point at time t is given by a function $x(t) = \dfrac{t^3}{3} - 5t^2$. At what times is its velocity equal to zero?

/3

Problem 6(b): At what times is its *acceleration* equal to zero?