Name: _____ GRADE: /50

Student #: _____

Okanagan University College
Salmon Arm Campus

MATH 112 – Calculus I

FINAL EXAM

9 December 2004

Instructor: Richard Taylor

Time allowed: 180 minutes

Instructions:

1. *Read all instructions carefully.*
2. *Read the whole exam before beginning*; make sure you have all 8 pages.
3. Organize and write your solutions neatly. If you run out of room, continue your solution on the back of the page.
4. Where appropriate, show your work and explain your solution method—a correct final answer alone is not sufficient to guarantee full credit. Part marks may be awarded even if you don't obtain the final answer.

Problem 1: Differentiate each of the following. *Please* don't simplify your answers.

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(a) $f(x) = (7x + 10)^{12}(8x^2 + 3x - 6)^{15}$

(b) $f(x) = (\sin^{38} x)\cos(38x)$

(c) $f(x) = \ln\sqrt{\dfrac{3x+6}{3x-6}}$

(d) $f(x) = \dfrac{x}{\arctan x}$

(e) $f(x) = xe^{-1/x}$

Problem 2: Evaluate the following limits (if they exist):

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(a) $\lim_{x \to 0} \dfrac{(9+x)^{-1} - 9^{-1}}{x}$

(b) $\lim_{x \to \infty} x^2 e^{-x/1000}$

(c) $\lim_{x \to \infty} x^{1/x}$ (*hint: take a logarithm first*)

Problem 3: (a) State the definition of the derivative of a function $f(x)$.

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(b) Use your definition from part (a) to find the derivative of $f(x) = x^3$.

Problem 4: Find an equation for the tangent line to the graph of

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$$\frac{x}{y} + \left(\frac{y}{x}\right)^3 = 2$$

at the point $(-1, 1)$.

Problem 5: Two methods of estimating $\sqrt{4.1}$...

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(a) Use an appropriate second-degree Taylor polynomial (i.e. $P_2(x)$) to estimate $\sqrt{4.1}$.

(b) Estimate $\sqrt{4.1}$ by using Newton's method to solve $x^2 - 4.1 = 0$, with "initial guess" $x_0 = 4$.

(c) Which method do you prefer? (Just out of curiosity)

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Problem 6: A police radar gun measures the rate of change of distance between itself and the object at which it is aimed. Suppose a radar gun is aimed in a direction making an angle 30° with a straight road. A car passes on the road, and the radar gun reads 115 km/h. How fast is the car going?

Problem 7: BONUS 2 MARKS:

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(a) Find both complex roots of $x^2 - 6x + 13 = 0$.

(b) Find $\sqrt{i}$, expressed as a complex number.

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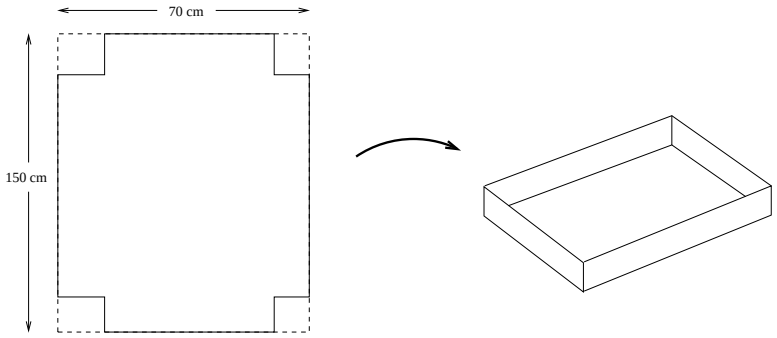
Problem 8: A box is to be made from a rectangular sheet of cardboard 70 cm by 150 cm, by cutting equal squares out of the four corners and bending up the resulting four flaps to make the sides of a box. (The box has no top.) What is the largest possible volume of the box? Justify your answer carefully (perhaps with appropriate use of the First Derivative Test?).



Problem 9: Sketch the graph of

$$y = \frac{1}{4 + x^2}.$$

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Pay particular attention to concavity and intervals of increase/decrease. Label all local maxima, minima, and inflection points on your graph.