Name: ____________________

GRADE: /57

Student #: ________________

Okanagan University College

Salmon Arm Campus

MATH 112 – Calculus I

FINAL EXAM

16 December 2003

Instructor: Richard Taylor

Instructions:

1. *Read all instructions carefully.*
2. *Read the whole exam before beginning*; make sure you have all 9 pages.
3. Organize and write your solutions **neatly**. If you run out of room, continue your solution on the back of the page.
4. Where appropriate, show your work and explain your solution method—a correct final answer alone is not sufficient to guarantee full credit. Part marks may be awarded even if you don't obtain the final answer.

Problem 1: Evaluate the derivatives of the following functions. It is not necessary to simplify your answers.

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(a) $f(x) = x^2 \ln x$

(b) $g(x) = \dfrac{x + \sin x}{1 + \cos x}$

(c) $F(x) = \arctan\left(\dfrac{1}{x}\right)$

(d) $G(x) = \tan\left(\sqrt{1 + e^{3x}}\right)$

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Problem 2: Evaluate the following limit:

$$\lim_{x\to 3} \frac{x^2-9}{x^2-x-6}$$

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Problem 3: Evaluate the following limit:

$$\lim_{x\to 1} \frac{\sqrt{x}-x^2}{1-\sqrt{x}}$$

Problem 4: State the *definition* of the derivative of $f(x)$.

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Problem 5: Use your definition from the previous problem to show that if $f(x) = \sqrt{x}$ then $f'(x) = \frac{1}{2\sqrt{x}}$.

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Problem 6: Find an equation for the tangent line at $(3, 1)$ to the curve

$$2(x^2 + y^2)^2 = 25(x^2 - y^2).$$

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Problem 7: Find all points on the curve $y = x^2$ where the tangent line passes through the point $(1, 0)$.

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Problem 8: Use Newton's method to find a positive solution of

$$e^x = 2 - x^2$$

accurate to 3 decimal places.

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Problem 9: (a) Use a linear approximation to estimate the value of $\sqrt{35}$. Give your answer to 5 decimal places.

(b) Use a second-degree Taylor polynomial to find an even better approximation of $\sqrt{35}$.

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Problem 10: Find the absolute maximum and minimum values of the function

$$f(x) = xe^{-x}$$

on the interval $[0, 2]$.

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Problem 11: The diagram below shows a right-angled triangle with base length 10 cm and height x. Suppose x is measured and found to be 4.0 cm, with uncertainty 0.1 cm. Find the angle θ and estimate the uncertainty in this value.

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Problem 12: A $500\,\text{cm}^3$ cylindrical soup can is to be designed so that its total surface area (including both ends) is as small as possible. What should be the dimensions of the can? Use the First Derivative Test to justify your solution.

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Problem 13: The trunk of a tree can be approximated by a cone ($V = \frac{1}{3}\pi r^2 h$). Suppose a particular tree has height 50 m and base radius 30 cm. If the tree is growing so that its base radius increases at a rate of 1.0 cm/year and its height increases at 40 cm/year, at what rate is the volume of its trunk increasing?

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Problem 14: A ladder 10 ft long rests against a vertical wall, with the base of the ladder 3 ft from the wall. If the bottom of the ladder slides away from the wall at 2 ft/s, how fast is the angle between the ladder and the wall changing?